

Lab 6: The Mass of Jupiter

Part 1: Introduction

In this lab, you will use astronomical observations of Jupiter and its satellites to measure the mass of Jupiter. We will use the program Stellarium to simulate observations of the night sky. By watching the orbits of Jupiter's moons, we will determine their orbital properties. These can be used to calculate the mass of Jupiter.

$$P^2 = a^3$$

Background material:

From your Introduction to Astronomy class, you have learned Kepler's laws of motion. In particular, the third law is presented as:

In this formula, P is the period, or the time for the orbiting body to complete one orbit, and a is the semi-major axis of the orbit. The law holds as written for particular units based on Earth: P in Earth years and a in Astronomical Units, or A.U., which are defined such that 1 A.U. is the average distance between the Earth and the Sun.

Question 1:

How would you determine the period of Jupiter's orbit using Kepler's third law? What information would you need?

Question 2:

How would you determine the semi-major axis of Jupiter's orbit using Kepler's third law? What information would you need?

$$F = \frac{GMm}{r^2}$$

$$G = 6.6737 \times 10^{-11} \frac{m^3}{kg s^2}$$

Kepler was able to accurately predict the motion of the planets that he observed, but Kepler's laws do not explain why this relationship holds. Later, Newton developed laws of motion and

gravity that could be used to predict Kepler's laws. In particular, he proposed Newton's law of gravity, which determines the force between two bodies:

In this formula, F is the force, M and m are the masses of the two bodies, and a is the distance between the two bodies. G is the gravitational constant, and is equal to

The value of G can be measured on Earth by very carefully measuring how massive objects influence each other. The uncertainty in our current best measurement of G is plus or minus $0.00080 \times 10^{-11} \text{ m}^3 / (\text{kg s}^2)$. Once we measure G on Earth, we can use the motion of objects in space to determine their masses.

$$P^2 = \frac{4\pi a^3}{G(M + m)}$$

Using Newton's law of gravity, and Newton's laws of motion, one can derive Newton's version of Kepler's third law:

You may notice that Newton's version of Kepler's third law uses the sum of the masses of the objects, but Kepler found all planets orbiting the Sun all have the same relationship between P and a , independent of mass. We can understand this because the Sun is much larger than the mass of any planets. Our best measurement mass of the Sun is $1.9885 \times 10^{30} \text{ kg}$, with an uncertainty of plus or minus $0.00035 \times 10^{30} \text{ kg}$. The mass of Earth, for example, is $5.972 \times 10^{24} \text{ kg} = 0.000005972 \times 10^{30} \text{ kg}$. Therefore the total mass of Earth plus the Sun is not measurably different from the mass of the Sun alone.

$$P^2 = \frac{4\pi a^3}{GM}$$

This is why all the planets appear to obey the same relationship: for systems where small bodies orbit much more massive bodies, we can ignore the mass of the small body. The resulting version of Kepler's law that we will use in this lab is:

Newton's laws of motion apply to all orbiting bodies, so we can use this relationship for other systems where small objects orbit a large orbit: planets orbiting other stars, moons orbiting other planets, and even stars orbiting the central mass of their galaxy.

Question 3:

If we want to describe the orbit of Jupiter's moon Callisto using Newton's universal version of Kepler's third Law, what would the variables represent?

M would represent

m would represent *the mass of the moon Callisto*

P would represent

a would represent

Finally, we can simplify this equation further if we use Earth orbiting the Sun as a reference for our units: The Earth orbits at $a = 1$ A.U. with a period $P = 1$ year around the Sun with $M = 1$ Solar mass. That means that, if we convert G from meters, kilograms, and seconds to A.U., Solar masses, and years, we must end up with a number that gives

$$\frac{4\pi}{G} = 1 \times \frac{(\text{Solar Mass})(\text{year})^2}{(\text{A.U.})^3}$$

With these units the form of Kepler's law that we will use in this lab is

$$(P \text{ in years})^2 = \frac{(a \text{ in A.U.})^3}{M \text{ in Solar masses}}$$

Question 4:

How would you rearrange the final form of Kepler's third law so that you can use it to find the mass of Jupiter from the period and semi-major axis of a Jovian moon orbit?

Hint: Rearrange the version of Kepler's third law so that the mass of the large body, M , is isolated on the left hand side of the equation.

Part 2: Using Stellarium

Starting Location, Date, and Time

Use the following steps to prepare for data collection.

- A) Find the shortcut to the Stellarium program on the computer and start the program.



- B) If the location in the bottom bar is not Fullerton: find Location in the left toolbar and click to open Location selection. Type Fullerton into the search bar and click on “Fullerton, United States.” Click “use as default” and close the Location window.



- C) Click the play/pause button in the bottom toolbar to stop the normal flow of time in Stellarium. The time in the bottom bar should stop changing.



- D) Open the Date and Time window from the left toolbar. Change the date to 2013-10-21. Then, change the time to 00:00:00 (midnight).



- E) Find Search in the left toolbar and open Search. Type in Jupiter, and the view will scroll to center on Jupiter. You can also use the arrow keys to change the camera direction.
- F) The “page up” and “page down” keys (or the scroll button) can be used to zoom in and out on the sky. The resulting field of view (FOV) is shown in the bottom bar - this is the angular distance across the window (top to bottom) in degrees. Zoom in until the stars Castor and Pollux (the “twins” in the constellation Gemini) are visible near Jupiter, at a FOV of approximately 20 degrees.

Measuring angular distances in Stellarium:

We'll take our angle measurements using the AngleMeasure plugin:

- A) Enable the tool by clicking the tool-bar button, or by pressing control-A. A message will appear at the bottom of the screen to tell you that the tool is active.
- B) Drag a line from the first point to the second point using the left mouse button. A measurement in degrees (d), arcminutes (m), and arcseconds (s) will appear.
- C) To clear the measurement, click the right mouse button.
- D) To deactivate the angle measure tool, press the tool-bar button again, or press control-A on the keyboard.

To test out the angular measurement tool, you will make some measurements of Jupiter and the surrounding stars.

Question 5:

What is the angular separation between Castor and Pollux on 2013-10-21 at 00:00:00?

Question 6:

What is the angular separation between Pollux and Jupiter on 2013-10-21 at 00:00:00?

- E) The “[” and “]” keys can be used change the date forward and backward by one week, while keeping the direction of view and the time of day the same. Use these keys to advance the date by three weeks to 2013-11-11. The stars and Jupiter are now higher in the sky at Midnight.

Question 7:

What is the angular separation between Castor and Pollux on 2013-11-11 at 00:00:00?

Question 8:

What is the angular separation between Pollux and Jupiter on 2013-11-11 at 00:00:00?

Question 9:

Did the angular separation between the stars Castor and Pollux change over the 3 weeks? Why or why not?

Question 10:

Did the angular separation between the star Pollux and the planet Jupiter change over the 3 weeks? Why or why not?



- F) Turn of angle measure and select Jupiter again. Switch to RA/dec mount, which will be better for tracking the motion of Jupiter through the sky.
- G) The “/” and “\” keys can be used to zoom in and out from the object of focus. Zoom in until you can see the Galilean satellites of Jupiter: Ganymede, Io, Europa, and Callisto. (One or more of these satellites may be obscured by Jupiter itself).
- H) Restart the flow of time with the play/pause button. At normal speed (1 second per second), nothing much will happen.



- I) Click the “fast forward” button to increase the speed of time in Stellarium. After a few clicks, you should be able to clearly see the motion of the moons over the course of the evening. When the sun rises, the sky turns blue (and a real astronomer would no longer be able to see Jupiter). When Jupiter sets below the horizon you will see the ground instead of the sky. After a couple days, press the play/pause button to halt the simulation.

Part 3: Measuring the motion of Jupiter's moons

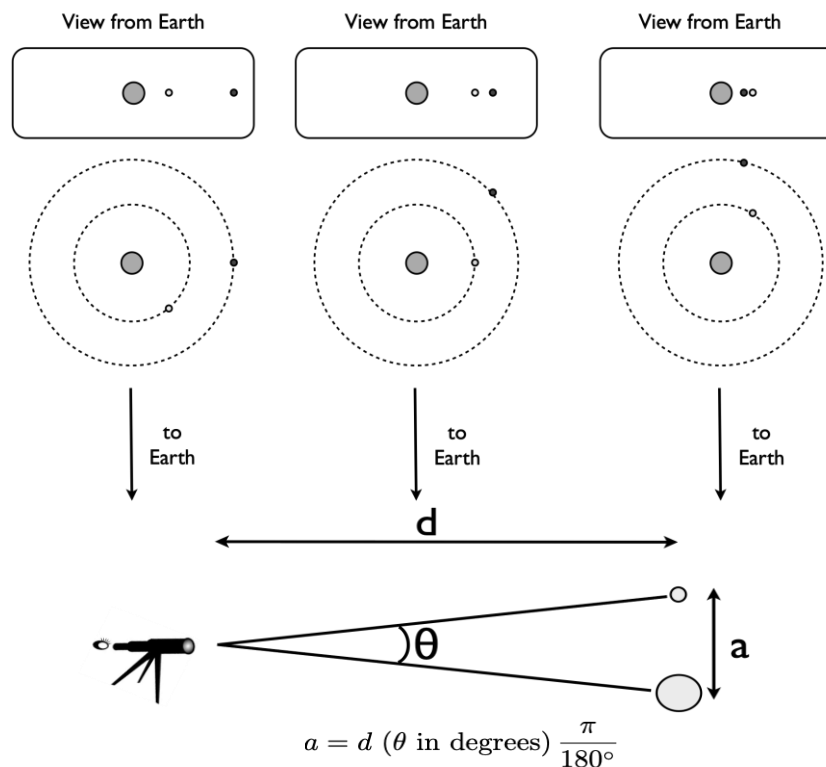
The orbits of Jupiter's moons are roughly in the same plane as the orbits of the planets around the Sun. Seen from Earth, the orbital motions of Jupiter's moons make them appear to travel side-to-side:

Angular distance and true distance:

When we observe objects in the sky, we measure their size or separation in terms of the angle between them. This angle depends on the distance to the objects.

When the moons of Jupiter reach their farthest angular separation from Jupiter, they are at a distance away from Jupiter which is approximately equal to the semi-major axis of their orbit, a . If we know the distance from Earth to Jupiter, d , we can measure the maximum angular separation θ . We then can calculate a using the following formula:

The calculated value of a will be in the same units as d , and we will need to use θ measured in degrees (as noted in the formula).



We'll find the value of a using the following steps.

- A) The distance to Jupiter must be determined to calculate the size of the moon orbits. Historically, this would be determined in A.U. by the Keplerian model of the solar

system; for our Stellarium observations, you can read the distance to Jupiter off the top left.

Question 11:

What is the distance to Jupiter on 2013-10-25 at 0:00:00;

$d =$

for this lab, we will approximate this as a constant for the measurements considered.

Next we need to find the maximum angular separation θ . We'll do so by taking some data over time.

Normal ground-based astronomical observations are restricted by the time of day and the orientation of the Earth, and the motions of Jupiter's satellites must be pieced together from many evenings of observation. In this lab, we will simplify things by ignoring these effects to make periodic observations of Jupiter's moons even during the day.

- B) In Stellarium, turn off the Ground and the Atmosphere by pressing G and A, or by clicking the icons in the text bar.



- C) Set the date to 2013-10-24 and the time to 00:00:00 (midnight). Make sure time is paused; you can use the Date/time window to control the time.
- D) We will watch the inner two moons of Jupiter, Io and Europa. They move quite quickly, so we will want to record their positions every four hours starting from this time. This data was mostly collected for you in the accompanying excel sheet but you'll use the following two questions to finish the data.

Question 12:

Measure the angular separation of the center of Io and the center of Jupiter at 00:00:00 (midnight) 10/24/2013 using the angular separation tool and note whether it is East or West of Jupiter.

Measure the angular separation between the center of Jupiter and the center of the moon in minutes and seconds, rounded to the nearest arcsecond.

Note: You are looking South to see the planet move near the Ecliptic plane, so East is to the left of Jupiter, and West is to the right.

Question 13:

Move forward in time 12 hours and measure the angular separation of the center of Io and the center of Jupiter at 12:00:00 (noon) 10/24/2013 using the angular separation tool and note whether it is East or West of Jupiter.

Measure the angular separation between the center of Jupiter and the center of the moon in minutes and seconds, rounded to the nearest arcsecond.

Note: You are looking South to see the planet move near the Ecliptic plane, so East is to the left of Jupiter, and West is to the right.

- E) Open the accompanying excel sheet "MassofJupiter-..." you'll see two days' worth of data has been collected for you. Insert the two values you just took into the appropriate missing boxes.
- F) Repeat the process in questions 14 for any other missing values in the Motions of the Moons table.
- G) Note the sheet automatically calculates the angle in decimal minutes for you, and includes a positive or negative based on the E, W location of the moon. There are also two graphs displayed that show the angular separation over time.
- H) Play out the rest of the time shown in the table in Stellarium, up to 10/26/2013 at 04:00:00. In particular pay attention to the motion of the two moons Europa and Io. Double check some other values from the sheet if you'd like.

Question 14:

Did Io make a full orbit of Jupiter, returning to its original position, in this time? Did Europa?

Part 4: Finding the Semi-Major Axis

Based on the excel data/graphs or your observation in Stellarium answer the following:

The Size of the orbits

Question 15:

Observe the data in the excel table. What was the largest angular separation (in decimal degrees) between Jupiter and Io in this time period? Of Jupiter and Europa?

Question 16:

These maximum angle separations may be a bit off from the true maximum. Explain how and how much effect you think it will have on your final measurements.

To calculate the semi-major axis of one of Jupiter's moons, remember the following formula. Here a is the semi-major axis, d is the distance to Jupiter, and θ is the maximum angular separation between Jupiter and the moon.

$$a = d \times \theta \times (\pi/180^\circ)$$

Both a and d will be in units of A.U. and θ needs to be in degrees. Remember 1 degree = 60 arcmin = 3600 arcseconds.

Question 17:

Insert the proper values into the small table "Semi-Major axis Calculation" in the excel sheet. This will use the equation above to calculate your semi major axis estimate.

What are your estimates of the semi-major axis for Io and Europa?

Io:

Europa:

Part 5: Finding the Period of the orbits

The period of an orbit is often labeled on graphs as the time between maximum angular separations on one side (e.g. W, or positive). However, it is easier to make a precise estimate using the zero-crossing times, when the angular separation goes from positive to negative as the moon appears to move past Jupiter.

On the excel graphs this means when the line crosses the x axis from positive to negative or negative to positive. The graphs shown have a smooth line connecting the data points so you should be able to estimate the crossing

Note: Remember a full period is going all the around a full circle, so the time between two zero crossings is half of a full period.

You'll find a table in the excel sheet called "Hours to Period" to assist you in this calculation.

Question 18:

Write down the apparent zero-crossing times for Io.

What is your best estimate of the period of Io in hours? What is this in years?

Question 19:

What are the zero-crossing times for Europa?

What is your best estimate of the period of Europa in hours? What is this in years?

Part 6: Finale, The Mass of Jupiter

Recall the formula for the mass of Jupiter that you found in Question 5 and use it with your a value in A.U. and period P value in years.

Remember this equation calculates your mass of Jupiter in “Solar Masses”, so multiplying your result by the mass of the sun will give you the mass of Jupiter in kg.

You’ll find a table in the excel sheet titled “Mass of Jupiter to assist you in these final calculations.

Question 20:

What is your estimate of Jupiter’s mass using the orbit of Io?

Question 21:

What is your estimate of Jupiter’s mass using the orbit of Europa?

Question 22:

Do your answers for questions 20 and 21 agree? Suggest possible reasons for any discrepancies.