Lab 1: Graphical Analysis

Experiment for Physics 211 Lab at CSU Fullerton.

What You Need To Know

Introduction

In this first lab for Physics 211 you won't be doing any physics. This statement may have just brightened your day, however, the ideas that you will be covering today are ones that students typically find difficult to master. This lab will take them a step at a time and hopefully you will leave today feeling Physics 211 Lab is not the worst thing in the world.

Analyzing A Graph

Sometimes experiments are performed to check the validity of an established theory for which an equation is already known. One way to accomplish this is to determine one of the *constants* in the known equation. This task can be simplified by using *graphical analysis*. This lab will go through a 4-Step process to explain this procedure. For each Step an example with fake data will be discussed first and then you will work through the same procedure using your own data.

What You Need to Do

Step 1 – Linearizing An Equation

Introduction

Graphical analysis will allow us to determine a *constant* by finding the slope of the line in a graph. Sometimes the plot will be linear and it is easy to find the slope. Other times the plot will be a curve and we cannot find a slope. So, the first step in this process is making sure that the plot is a straight line, i.e. linearizing your equation.

To explain this idea, we are going to use *two examples* based on the "equation of state" for an ideal gas …

$$PV = RT$$

P is the pressure (in atmospheres, atm)

V is the volume (in liters, ℓ)

T is the temperature (in Kelvin, K)

R is the ideal gas constant (in L-atm/K)

Equation 1 – Ideal Gas Law

At the end of the 4-Step process we will determine the ideal gas constant, ***R***, and see how it compares to the established value of 0.0820 ℓ-atm/K.

Example 1

In the first example, let's say you are going to perform an experiment in which we vary the *temperature* and then measure the *pressure* while holding the *volume* constant. This means that the *independent* variable (the value we control) is the temperature ***T***. The *dependent* variable would then be the pressure ***P***. Both ***R*** and ***V*** are constants. We would then run the experiment and collect data for ***T*** and ***P***. Before plotting the data, we must make sure that we have a linear relationship between the independent and dependent variables. In other words, we need to see if the equation is in linear form: $y = mx + b$.

The ***y*** part of the linear form must involve the dependent variable and the ***x*** part must involve the independent variable.

Linear Form	Must Involve
y part	dependent variable
x part	independent variable

Figure 1 – Independent and Dependent Variable

We will need to rearrange or "massage" the equation so that ***y*** matches up with ***P*** and ***x*** matches up with ***T***.

A) Let's start by dividing both sides of the equation by ***V***. This will leave ***P*** by itself on the left side, just like ***y*** is by itself in the linear form. See **Figure 2.**

B) The ***RT/V***, however, doesn't quite match with the ***mx*** part and there is nothing to match up with ***b***. So, we rearrange the fraction a little as well as adding a **0**. **See Figure 1b.**

C) Now the two equations match. The ***y*** (y-axis) is equivalent to the ***P***, the ***m*** (the slope) is equivalent to the ***R/V***, the ***x*** (x-axis) is equivalent to the ***T***, and the ***b*** (the y-intercept) is equivalent to the **0**.

a)	b)
$P = \frac{RT}{V}$	$P = \left(\frac{R}{V}\right)T + 0$
↓ ↓	↓ ↓ ↓ ↓
$y = mx + b$	$y = (m)x + b$

Figure 2 – Step 1 Example 1

Example 2

In this *second example*, we are going to use the same equation but now we will change the experiment and vary the *pressure* and then measure the *volume* while holding the *temperature* constant at 350 K. This means that the *independent* variable is the pressure ***P***. The *dependent* variable would then be the volume ***V***. Both ***R*** and ***T*** are constants. We'd then run the experiment and collect data for ***P*** and ***V***.

We need to go back to the original equation so that it can be massaged into linear form.

A) In this example, the ***y*** part needs to involve ***V*** since it is the dependent variable and the ***x*** needs to involve ***P*** since it is the independent variable. **Figure 3** shows the algebraic procedure for this. The process is similar to what we did before but it is different in how we rearranged the ***mx*** part.

B) Now, the ***y*** (the y-axis) is equivalent to the ***V***, the ***m*** (the slope) is equivalent to the ***RT***, the ***x*** (the x-axis) is equivalent to the ***1/P***, and the ***b*** (the y-intercept) is equivalent to the **0**.

a)	b)
$V = \frac{RT}{P}$	$P = (RT)\frac{1}{P} + 0$
↓ ↓	↓ ↓ ↓ ↓
$y = mx + b$	$y = (m)\ x + b$

Figure 3 – Step 1 Example 2

Before moving on to Step 2 in the process you are going to apply what you have learned so far. In each of the problems below, a theoretical equation is given and the variables and constants are identified. In some cases you are told which variable is to be the independent one and which is to be the dependent one. In other cases it is up to you to decide.

Exercise #1

Do the following for each of the following questions

a) "massage" the equation into linear form
b) identify in the linearized equation your ***y***, ***m***, ***x***, and ***b***

EXAMPLE

The impedance of a series RC circuit is given by:

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega^2 C^2}\right)}$$

Z is the dependent variable and ***ω*** is the independent variable. ***R*** and ***C*** are constants.

a)

1. $Z = \sqrt{R^2 + \left(\frac{1}{\omega^2 C^2}\right)}$

2. $Z^2 = R^2 + \left(\frac{1}{\omega^2 C^2}\right)$

3. $Z^2 = \left(\frac{1}{C^2}\right)\left(\frac{1}{\omega^2}\right) + R^2$

b)

$$Z^2 = \left(\frac{1}{C^2}\right)\left(\frac{1}{\omega^2}\right) + R^2$$

↓ ↓ ↓ ↓

$$y = m \quad x + b$$

Now Your turn

Question 1:

The resonant frequency, ω, of a parallel L-C circuit is given by:

$$\omega = \frac{1}{\sqrt{LC}}$$

ω and C are measured variables, where ω is the independent variable, C is the dependent variable, and L is held constant.

Question 2:

The linear expansion of a solid, ℓ, is described by:

$$\ell = \ell_0(1 + \alpha\Delta T)$$

$\boldsymbol{\ell}$ and $\Delta T$are measured variables. $\ell_0$and α are constants.

Question 3:

A conical pendulum has a period, T, given by:

$$T = 2\pi\sqrt{\frac{l(cos\theta)}{g}}$$

T and θ are measured variables, where θ is the independent variable. l and g are constants.

Question 4:

The wavelength, λ, of the light in the Balmer series of the spectrum of the hydrogen atom is given by:

$$\frac{1}{\lambda} = B\left(\frac{1}{4} - \frac{1}{n^2}\right)$$

λ and $\boldsymbol{n}$ are measured variables, where n is the independent variable. B is a constant known as the Rydberg constant.

Step 2 – Determining Axes and Modifying Data Table

Introduction

We are now going to go back to our *examples* from Step 1. We massaged our equation into linear form and now we need to determine how to plot our data, in other words, we need to determine what to plot on the x-axis and the y-axis. By looking at the linearization results, we will see how to do this.

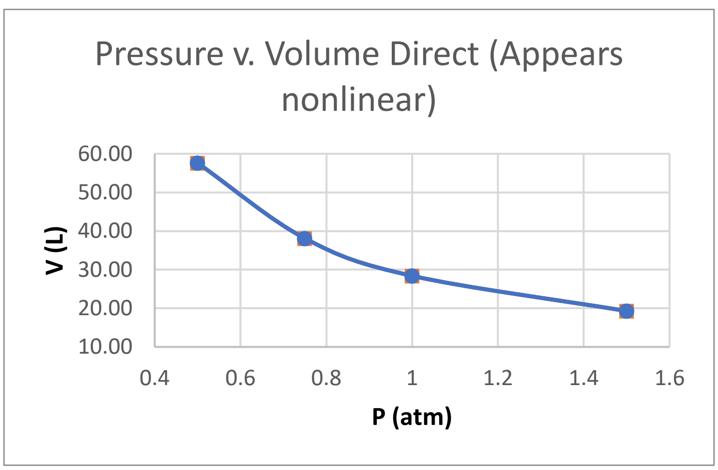
Example 1

Let's go back and look at the results of the *first example* we did. See *Figure 2*. We determined that on the y-axis we are going to graph the pressure data. On the x-axis we are going to graph the temperature data. So, we would make a graph of ***P*** vs. ***T***. The plot would be linear and we can calculate a slope.

Example 2

In the *second example*, we determined that on the y-axis we are going to graph the volume data and on the x-axis we are going to graph "1 divided by the pressure data", ***1/P***. See *Figure 3*. If we tried to graph the data as simply ***V*** vs. ***P***, the plot would come out as a curve (i.e. non-linear) and we would not be able to calculate a slope.

To show that let's say we took data based on the *second example* we've been discussing, the one in which we varied the *pressure* and then measured the *volume* while holding the *temperature* constant. Plotting that data directly gives us a nonlinear graph as shown.

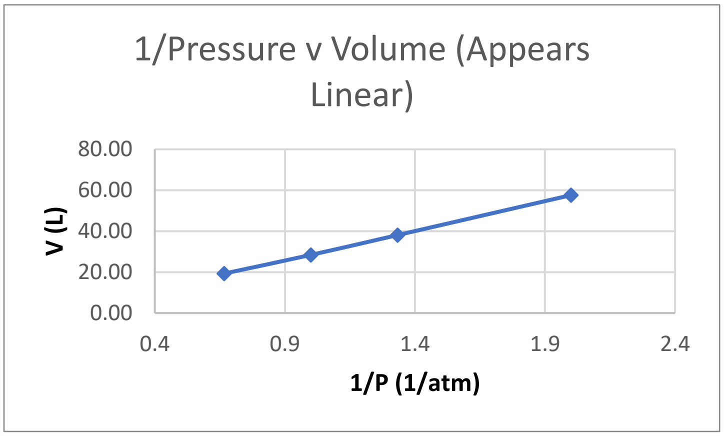


Variables	
P (atm)	**V (L)**
0.50	57.6
0.75	38.1
1.00	28.4
1.50	19.3

Figure 4 -Step 2 Example 2 Raw

So, in order to get a straight line we must make a graph ***V*** vs. ***1/P***. This process is also known as "forcing a line" in order to "force" the curved line to become a straight line.

We need to make another column with a title of ***1/P***. We would then take each of the pressure data and divide them into 1, putting these new values into an additional column. This will allow us to make a graph of ***V*** vs. ***1/P*** which will give us a linear plot as shown below.



Variables		
P (atm)	**1/P (1/atm)**	**V (L)**
0.50	**2.00**	57.6
0.75	**1.33**	38.1
1.00	**1.00**	28.4
1.50	**0.67**	19.3

Figure 5 – Step 2 Example 2 Linearized

In this second *example* only the x-axis needed modification, but there are equations in which you will need to modify BOTH of your axes in order to get a linear plot.

Exercise #2

You are now going to do the same process described above but you will be using the equation and results from **Question 4.** For your data you will use the data in **Table 1**. Do the following Questions…

Table 1 – Step 2 Exercise 2

Variables		
n	λ (m)	
3	6.56×10^{-7}	
4	4.86×10^{-7}	
5	4.34×10^{-7}	
6	4.10×10^{-7}	
7	3.97×10^{-7}	

Question 5:

Based on your results from **Problem #4** in **Exercise #1**, state how you are going to graph your data on the x-axis and the y-axis.

Question 6:

Based on your answer to Question 5, modify your data table (like in the *example* in **Figure 3b)** so that when you make your graph you will get a straight line. Do not leave your data in fraction form. Use a calculator or excel to get a decimal number.

Step 3 – Plotting Your Data

Introduction

Computers are very handy tools. Sometimes, however, they make things too easy for students and they never learn some basic skills. For example, many students don't have a clue how a trendline is made or what it even means.

In this section you will create a graph and best fit line by hand, so that when you click "Add Trendline" or "Linear Fit" in later labs its more clear what you're doing and why.

There are two main ideas to keep in mind while plotting data by hand. They are …

I. **Use more than half of the graph paper.**

II. **Make your data easy to plot.**

In order to make a proper graph we need to strike a balance between these ideas. To demonstrate this we are going to go through an *example* using the *fake data* in Figure 6.

Figure 6 – Step 3 Fake Data

X	Y
5	920
12	1130
20	1610
32	2180
44	2730
51	3300
63	3960

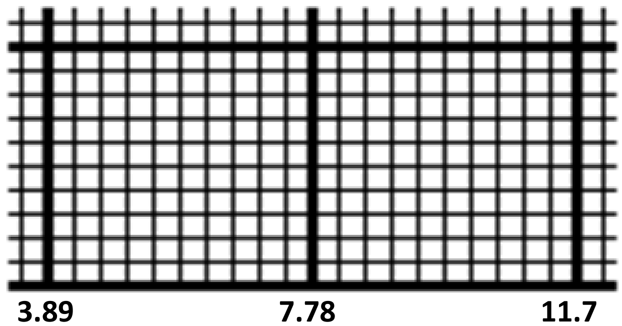
Before actually plotting the fake data in this *example* we need to assign values on the horizontal and vertical axes. Let's examine the data. The X data has a minimum of about 0 and a maximum of about 70. Therefore, on one axis we will spread out the assigned axis values from 0 to 70. The Y data has a maximum of about 4000, however, we wouldn't want to assign the axis values from 0 to 4000. Since there is no Y data from 0 to 920, we wouldn't be using that region of the graph paper. We should then only assign values from roughly 900 to 4000.

Next, we need to decide on the orientation of the graph paper. The graph paper used in this lab has 18 by 25 major divisions each with 10 subdivisions. (Examine the graph paper at your table.) The X data gives a spread of 70. The Y data gives a spread of 3100 … (4000 – 900 = 3100). Typically, one chooses to put the data that has the largest spread on the axis that has the most divisions. So, in this case, the Y axis will have 25 major divisions and the X axis will have 18 major divisions.

Ok, now we are going to assign the values for the divisions. Let's discuss some options for the X data.

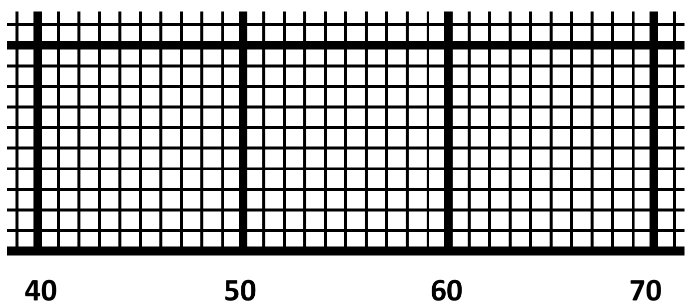
Option #1: we could take the 18 major divisions and divide that into the "spread" of 70 [70 / 18 = 3.89]. So, each major division would have values that are multiples of 3.89 and the subdivisions would have values of 0.389. **See Figure 7.** This certainly would make the graph very large but it would also make the data very difficult to plot. So, let's go to Option #2 …

Figure 7



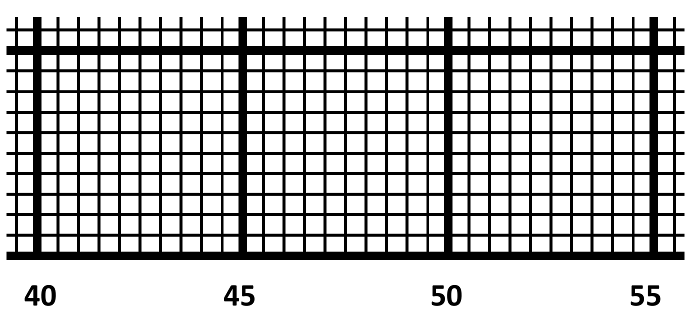
Option #2: We can make the data extremely easy to plot if we make each major division equal to 10 with subdivisions having a value of 1. **See Figure 8.** However, in examining the graph paper, we would be using less than half of the graph paper on the x-axis.

Figure 8



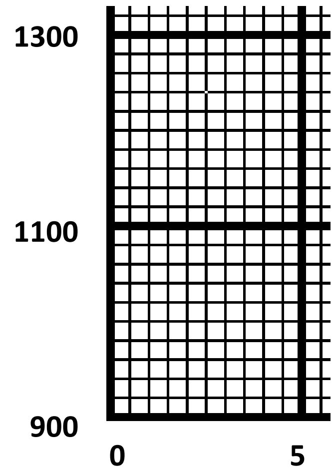
Option #3: The best option is to expand the axis by doubling the number of subdivisions in between the assigned values of 10. **See Figure 9.** This will accomplish two things. First, we will be using more than half of the graph paper (about 3/4 of it). Second, the data will still be fairly easy to plot with each major division having a value of 5. Each subdivision will then have a value of a half or, a better way to think of it, every two subdivisions will have a value of one.

Figure 9



In examining the Y data, we see that the numbers are quite large and simply assigning a value of 10 to the major divisions won't work. If we try multiplying by a factor of 10, making each major division worth 100, we won't have enough room with a spread of 3100 and only 25 major divisions. Since we now have the opposite problem as before, we can use the opposite solution. Instead of expanding the number of subdivisions we can contract them and as a result each major division is now worth 200. The subdivisions will then have a value of 20. **See Figure 10.**

Figure 10



Make Your Own Graph

Now do the same process as in Step 3 but you will be using the data you calculated in *your* modified data table from **Exercise #2 in Step 2.**. Determine how you are going to spread out your data for each axis and then plot it. You will be graded on how well you adhere to what was discussed in this section.

Question 7:

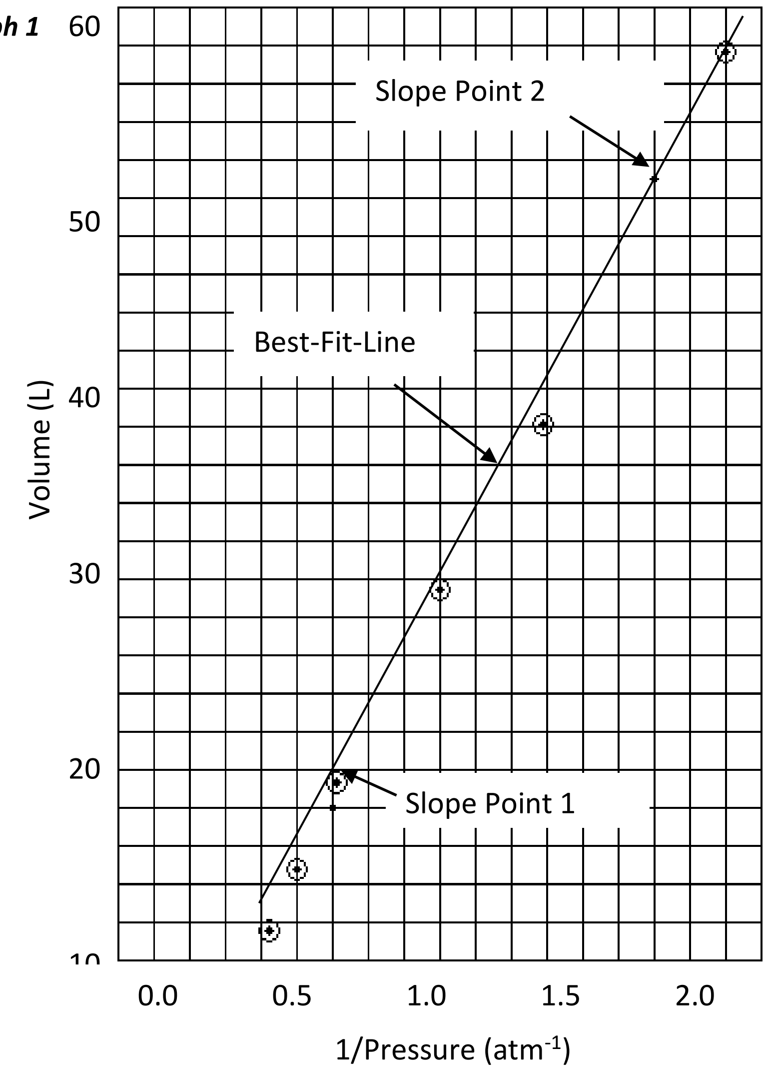
Plot the data from exercise 2 by hand on the graph paper provided by your instructor. Be sure to follow the instructions in step 3.

Step 4 – Finding Your Slope and Constant

Introduction

We are now going to go back to our *example* from Step 2. Now that we have made our graph we can then proceed to find the slope. As an *example* let's say we have graphed the data from the modified data table from Step 2 using the data in **Figure 4**. **See Graph 1.** Notice how the axes values were chosen so that a majority of the graph is used. Also notice the labeling of each axis with a name and the units.

Graph 1



The line in the graph is called a "best-fit-line" (BFL). The BFL is a line that best approximates the linear nature of your data. A BFL is drawn by visually equalizing the amount of deviation of the data points above and below the line. It is NOT a requirement that your data points actually fall on your BFL as in this case. However, if a data point does fall on the BFL, it is merely coincidence and it does not make that data point "better" than any other.

Let's calculate the slope of the BFL in this example. To do this, we need to choose two points on the BFL based on the following two criteria. First, we want the points to be far apart on the line. **See Graph 1.** This will give you a more accurate slope. The second criterion is to choose the points so that they are easy to read. In **Graph 1**, two slope points are shown. Notice that the **Slope Point 1** is easy to read because it falls right at an intersection of a horizontal and a vertical line. There isn't another point at an intersection so we should at least choose a point that falls on at least one line. See **Slope Point 2 marked on Graph 1**. The coordinates of these points, as well as the slope calculation, are given in **Figure 12.**

Slope Point 1 $(0.625\ 1/atm,\ 18\ L)$ Slope Point 2 $(1.75\ 1/atm,\ 51\ L)$	$m = \frac{Rise}{Run} = \frac{\Delta V}{\Delta\left(\frac{1}{P}\right)} = \frac{(V_2 - V_1)}{\frac{1}{P_2} - \frac{1}{P_1}}$ $m = \frac{51L - 18L}{1.75\frac{1}{atm} - 0.625\frac{1}{atm}}$ $m = 29.33\frac{L}{\frac{1}{atm}} = 29.33\ L \cdot atm$

Figure 11 – Sample slope calculation

The entire purpose of this procedure has been to find and verify a constant in our equation. Now we can finally find it. The constant for the equation we've been dealing with is the ideal gas constant, ***R***. The other constant, ***T***, we stated earlier would be held at 350 K.

From our *second example* in Step 1, we have already determined that the slope, ***m***, was determined to be equivalent to ***RT***. **See Figure 3**

The calculations are done below to determine the value of ***R*** and also the percent difference associated with its true value of 0.0820 L-atm/K. **See Figure 12**

$m = RT$ $R = \frac{m}{T}$ $R = \frac{29.33\ L \cdot atm}{350\ K}$ $R = 0.0838\ L \cdot atm/K$	$\%\ Difference = \frac{\lvert value_1 - value_2 \rvert}{average} \times 100$ $\%Difference = \frac{\lvert 0.0820 - 0.0838 \rvert}{0.0829} \times 100$ $\%\ Difference = 2.17\%$

Figure 12 - Determining R and % Difference

Exercise #4

You are now going to do the same process as in Step 4 but you will be using the graph you made in **Exercise #3.**

Question 8:

Start by drawing your BFL using the clear ruler on your table.

Based on your BFL, calculate the slope of the line.

Use your slope to calculate the Rydberg constant, ***B***, based on your linearized equation from **Problem #4** in **Exercise #1**.

Calculate a percent difference by comparing your calculated value to the actual value of 1.097×10^{7} m^{-1}.

Note on Percent Difference

When running experiments and taking data we are testing real world experiment against theoretical values. We don't want to assume that either is correct; we run the experiments to test expectations after all. So for this course whenever you're asked to compute a percent difference for the rest of the course, use the equation

$$\%\,difference = \frac{|value_1 - value_2|}{average} * 100$$

Step 5: Your Turn

Now you are going to do the entire 4-Step process over again using data that you will collect yourself. At your lab table you will find a ball hanging from a string of length, ***L***, a timer, and a meter stick. You are going to allow the ball to swing back and forth and then measure the period of this pendulum, ***T***.

The equation of the period of a pendulum is given by …

$$T = 2\pi\sqrt{\frac{L}{g}}$$

T and ***L*** are measured variables. ***g*** is a constant.

Using the procedure you learned in this lab, determine the constant, ***g***, whose value is $9.8\ m/s^2$. It is up to you to determine your independent and dependent variables. You should also calculate a percent difference.

This time find your best fit line using Excel…

Note MS office 365 is installed on our computers AND it is free to all CSUF Students. Use one of those options to make things easy, using other software like google docs is fine but will have different methods, so only use if you're confident.

A) Take data on the swinging pendulum with the following steps and enter your values into two adjacent columns in excel. Something like Figure X.

  a. Let the ball swing back and forth 10 times. Make sure that the swing of the ball is small, only about 15˚ to each side.

  b. Measure that entire time and then divide this time by 10. This will give you the period for one swing.

  c. Take data for at least 5 different lengths that vary from 15 cm to 90 cm. Use the black metal clip to change the length of the string. Put your data in the table in the report.

L(m)	T(s)

Figure 13 – Excel table

B) After massaging the equation for period into a linear form, add a column for the variable you're transforming. Note you can use excel equations to calculate the new linearized column to make things easy (try typing =sqrt()). You should now have three columns, see the following Figure.

L(m)	sqrt(L)	T(s)
0.15	0.387	0.74
0.3	0.548	1.11
0.45	0.671	1.38
0.6	0.775	1.55
0.75	0.866	1.75
0.9	0.949	1.88

Figure 14 – Excel table with sample data

C) Now make your graph with the following steps, but all the rules of a hand drawn graph from earlier still apply, make the data take the entire graph, pick good axis limits, axis and graph titles, etc.

a. Click and hold the first value in your x-axis column.

b. Drag to the bottom value of your y-axis column.

Note it helps to have x column first then y column since that's excels default order. You can switch them after you make the graph by clicking the data and moving the highlighted boxes, or rightclicking the graph, selecting "Select Data" and changing it there.

L(m)	sqrt(L)	T(s)
0.15	0.387	0.74
0.3	0.548	1.11
0.45	0.671	1.38
0.6	0.775	1.55
0.75	0.866	1.75
0.9	0.949	1.88

Figure 15 – Excel table highlighting

c. Once you have your two columns selected as shown, go to the top of the excel sheet and locate the "Insert" tab. (Office 365 word bar shown)

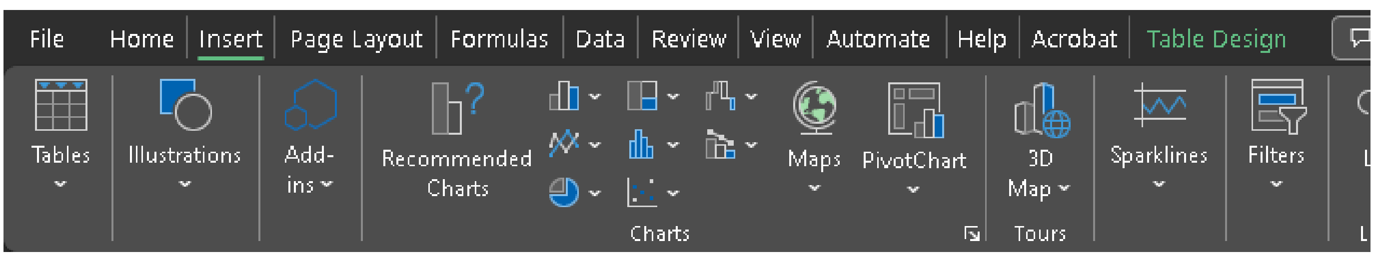
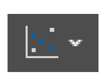


Figure 16 – MS Word top ribbon, showing insert

d. Then click the scatter chart icon  and select the first one, "scatter"

D) You should now have the simple scatter plot shown. From here you can adjust most everything by right clicking it, or adding missing things like axis labels from the + menu at the top right of the graph.

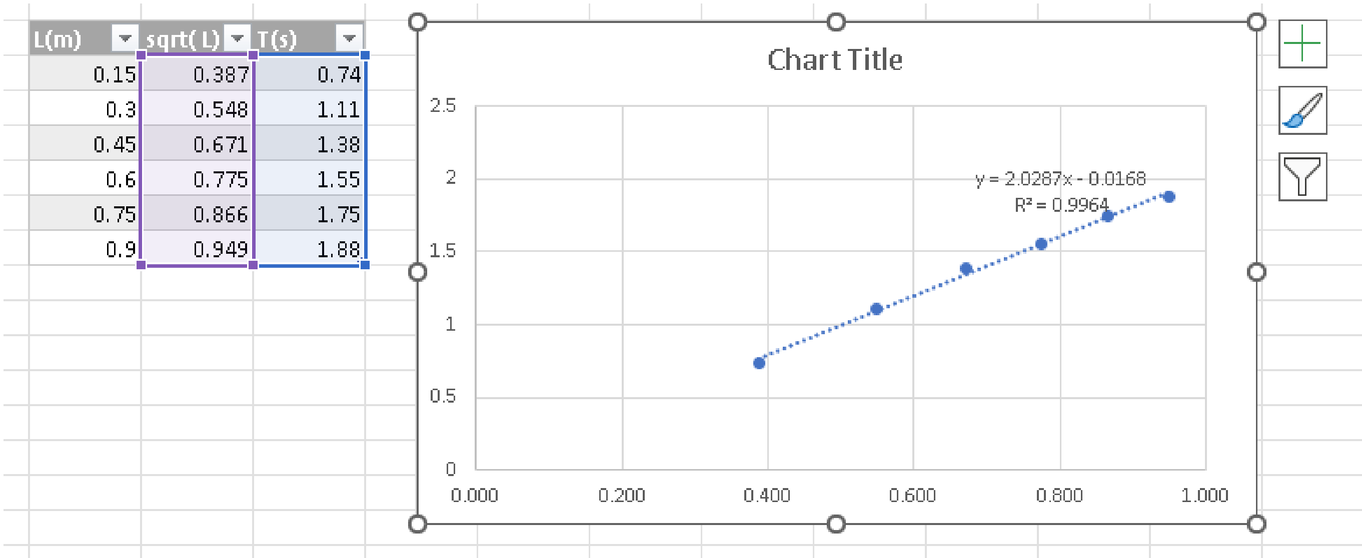


Figure 17 – Unformatted sample graph

E) In particular we need a trendline for slope and y intercept.

a. In the + menu click the arrow next to trendline, then select “More Options” .

b. In the “Format Trendline” menu check the bottom two boxes, Display Equation and Display R-squared value.

Excel just did the same thing you did by hand, but with a bit more precision and mathematics. A nice consequence is that we get the “R-squared” value. This value you tells you how close to the best fit line your points are on average, the closer to 1 the better, with 1 meaning every dot falls right on the line.

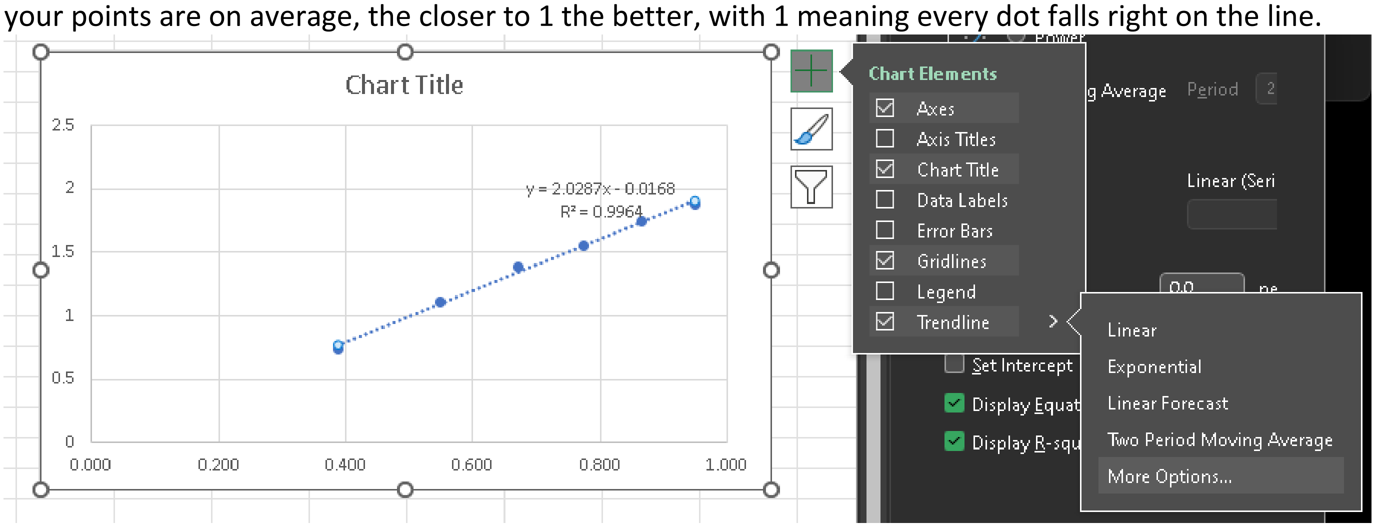


Figure 18 - + Menu in MS Word

Notice in the example here an R^2 of 0.9964. That's pretty good, probably because I used a calculator to make artificial values, then randomly adjusted the values just a little bit. We usually like 4 nines in our more precise labs here. All right finish with the last questions and conclusion.

Question 9:

Using the trendline (A.K.A. Best Fit Line) determine the constant, ***g***, whose value is 9.8 m/s^2.

Calculate a percent difference.

Question 10:

Adjust the graph based on what you've learned in a good graph should have. Note for excel graphs we also want axis titles and a good chart title. Submit your final polished graph and table by copying and pasting into your report. The table you can just select and ctrl-c, ctrl-v.

For your graph, when you paste it don't ctrl-v, right click and select the paste as picture option as shown in Figure X, this will avoid any formatting issues.

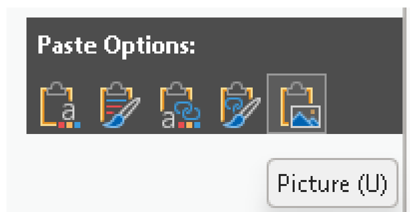


Figure 19 – MS Word Paste Menu

Conclusion

Follow the lab report guide to write a conclusion on this lab. Focus on just step 5 for this conclusion.

Conclusion