# Lab 12: Physical Pendulum

Experiment for Physics 225 Lab at CSU Fullerton

# What You Need to Know

# Introduction:

This experiment has two parts:

- 1. In part one you use 1 adjustable brass mass and a thin rod as a simple pendulum. The goal is to measure and calculate the period, as well as take data for large angles.
- 2. In part two you use 2 adjustable brass masses, one above and one below the axis of rotation, and one thin rod. Again you will measure and calculate the period of this pendulum.

# Part I: Single mass Pendulum

Newton's second law in rotational form is

$$I_p \alpha = \sum \tau \tag{1}$$

 $I_p$  is the moment of inertia about the pivot point, a constant value.  $\alpha$  is the angular acceleration and  $\tau$  are the torques on our system.

Remember that the angular acceleration is given by

$$\alpha = \frac{d^2\theta}{dt^2} \tag{2}$$

We will start with the pendulum shown in *Figure 1*,



Figure 1 – Single Mass Pendulum

Equation (1) can be written as

$$l_p \alpha = -(l_{cm}) \times (mg) \times \sin \theta \tag{3}$$

#### **Physical Pendulum**

#### By using (2) and simplifying the cross product this becomes

$$\frac{d^2\theta}{dt^2} = -\left(\frac{mgl_{cm}}{I_p}\right)sin\theta\tag{4}$$

This is our equation of motion for the pendulum with moment of inertia  $I_p$ . Now let's get it to make a bit more sense, first call the term on the right side in parenthesis  $\omega_0^2$  so that we have

$$\sqrt{\frac{I_p}{mgl_{cm}}} = \omega_0 \tag{5}$$

Also assume  $\theta$  is small so that we can use the approximation  $\sin \theta \approx \theta$ .

This leaves us with our new equation of motion

$$\frac{d^2\theta}{dt^2} = -\omega_0^2\theta \tag{6}$$

which can be solved to get the equation for *theta* as a function of time as

$$\theta = \theta_0 \sin(\omega_0 t + \phi) \tag{7}$$

This is an equation for simple harmonic motion, with a frequency  $\omega_0$  .(Also an amplitude  $\theta_0$  and phase  $\phi$ ) Here we are interested in the period of our pendulum, so from the frequency we have:

$$T_{calculated} = \frac{2\pi}{\omega_0} = 2\pi \sqrt{\frac{\left(l_1 + \frac{1}{12} \frac{m_{ROD} l_{ROD}^2}{m l_1}\right)}{g}}$$
(8)

Note that this is equivalent to the two mass case in part 2 (eqn. 11) with  $l_2$  taken as zero.

## Part II: Two masses

Now we will add a mass to the other side of the pendulum as shown in *Figure 2*.

The equation for the period is the same as part one, given by

$$T = \frac{2\pi}{\omega_0} = 2\pi \sqrt{\frac{l_p}{mgl_{cm}}}$$
(9)

The moment of inertia is similar to part one but the center of mass has moved, so our new equation is

$$\frac{1}{12}m_{rod}L^2 + m(l_1^2 + l_2^2) \tag{10}$$

This leaves us with the equation for the period as

$$T_{calculated} = 2\pi \sqrt{\frac{\frac{1}{12} \frac{m_{rod}}{m} l_{rod}^2 + (l_1^2 + l_2^2)}{g(l_1 - l_2)}}$$
(11)



Figure 2 – Double Mass pendulum

# **The Equipment**

To allow you to focus on the physics of the pendulum use the values given here as needed in your calculations. This way you don't have to take the system apart to make any measurements.

 $m_{rod} = 27.68g, (26.24g \text{ without screw})$   $m_{weight} = \{75.52g, 75.48g, 75.39g\}$   $l_{rod} = 38.0cm$   $l_{mass} = \{20.05mm, 20.06mm, 20.08mm\}$ 

## What You Need to Do:

## Part 1: Procedure

The goal for the first part of the experiment is to measure the period of a pendulum.

A) Make a copy of <u>*Table 1*</u> in your report to fill in for this part.

#### Table 1 – Part 1 Data

$m_{rd}  (kg)$	m (kg)	l <sub>1</sub> ( )	θ (deg)	$T_{calculated}$ (eqn. 8) ( )	T <sub>measured</sub> ( )	% error
.02768						
.02768						

B) Prepare the equipment for the measurement:

- a. First begin by insuring the rotary motion sensor is plugged into the dig/sonic 1 port. Then open the Physical Pendulum lab file.
- b. In order to get the logger pro to start reading from the rotary motion sensor unplug the cable from dig/sonic 1 and plug it into dig/sonic II. You should now see an angle being read, move the pendulum to ensure that it is reading.
- c. Now that the system is reading, stop the motion of the pendulum and then zero out the system.

NOTE: Whenever you press collect the system will rezero to where the pendulum currently is, so make sure the pendulum is at zero, hit collect, then move the pendulum to starting point.

- C) Put the brass mass on one end of the rod, near the end as shown. Measure the distance from the center of the pendulum to the edge of the mass, then add half of the height of the mass to this to get  $l_1$ .
- D) Measure the period for an angle  $\theta$  less than 15 degrees:
  - d. To measure the period, take a set of data then highlight ten full oscillations.
  - e. Take that total time for 10 oscillations, then divide that time by 10 in order to get the time per one oscillation (a.k.a the period).
- E) Calculate the period you expect from the small angle approximation, equation 8. Find the % error to your experimental value.
- F) Repeat C and D for an angle  $\theta$  larger than 50 degrees. See <u>Figure 3</u> for examples of how large the angles should be.

## Question 1:

Does the small angle approximation hold for this larger value of  $\theta$ ?



Figure 3 – Angle examples: 10 degrees on the left, 60 degrees on the right

## Part 2: Two Masses

Your goal for this section is to measure the behavior of the period as you add a second mass on the other side of the pivot at different locations.

A) Make a copy of *Table 2* in your report to fill in for this part.

Table 2 – Part 2 Data

$m_{rod}  (kg)$	m (kg)	l <sub>1</sub> ( )	l <sub>2</sub> ( )	T <sub>calculated</sub> (eqn. 11) ( )	T <sub>measured</sub> ( )	% error
.02768						
.02768						
.02768						

- B) Prepare the equipment for the first measurement:
  - a. The distance to the first mass  $l_1$  should be the same as in the first part of the experiment, fill this into <u>Table 2</u>.
  - b. Put the second mass on the other side of the pendulum as close to the center as you can. Measure  $l_2$  the same way as  $l_1$ , measure from the center to the inner edge of the mass and then add half the height of the mass to get  $l_2$ .
- G) Calculate the period using equation X for the two mass system.

H) Experimentally measure the period using the same method as before, then calculate your percent error for the system.

## Question 2:

How do you expect the period to behave as  $l_2$  approaches  $l_1$ ? Explain your answer.

- I) Move the second mass about half way out on the rod and then repeat steps B, C.
- J) Move the second mass further out toward the end of the rod, but so that  $l_2$  is still less than  $l_1$ . Repeat steps B, C.

# Question 3:

What is happening to the period of your system as I2 is increased? Does this agree with your answer from question 2?

# Conclusion

Follow the lab report guide to write a conclusion on this lab.

Submit any additional excel or graphical analysis data your instructor requests along with your report.

## Conclusion

Rev 2023