Lab

11

Simple Harmonic Motion (SHM)

What You Need To Know:

The Physics: In figure 1, the mass m is, at time t, displaced by a distance x (down is positive) from its equilibrium position. The spring force , of magnitude kx, is directed in the negative x direction (upward), so we have by Newton's 2^{nd} Law:

$$F = ma = m\frac{d^2x}{dt^2} = -kx \qquad (1)$$

The integrated equation gives,

$$x(t)=A\cos[2\pi ft+\theta] \qquad (2)$$

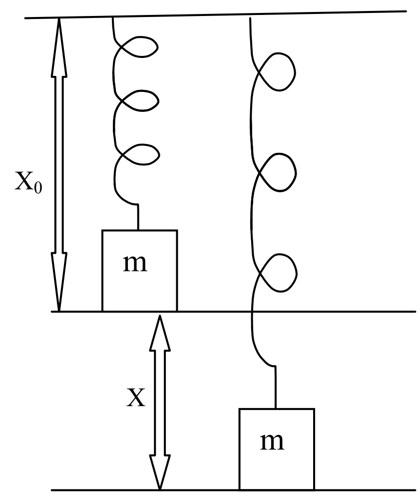
and $(2\pi f)^2=k/m$ with A and θ being arbitrary constants of the integration. These can be adjusted to give the desired values of x and f at time $t=0$. The period of the motion T, is defined such that $2\pi fT = 2\pi$, so that the mass goes all the way down and all the up and back to its starting position. Thus the period is one over the frequency in Hertz, $T = 1/f$.

The student should differentiate *eqn. (2)* twice and substitute into *eqn. (1)* to verify the solution works. Finally we have,

$$T = 2\pi\sqrt{\frac{M}{k}} \qquad (3)$$

where M = total effective mass.

Figure 1.



DO NOT OVER STRETCH THE SPRINGS- this will render them useless and the results will be very poor. The spring will no longer obey Hooke's law !

What You Need To Do :

The terminology:

x = displacement of mass m from equilibrium.
A = amplitude of the motion. The maximum distance the mass travels from its equilibrium position.
f = frequency of the motion in per second or Hertz.
T = the period, time for one complete cycle of the oscillation.
t = time measured from an arbitrary zero time.
θ = phase constant, measured in radians, determined by values of x and f at time t=0.

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(1) **Static measurement**: You are to verify Hooke's law, which holds for many stretchable and compressible objects, provided the deformation (stretching) of the object is not too great. Hooke's law, states that the amount of stretch
(or compression) of an object, x, is directly proportional to the force, F that produces the stretch (or compression). $F = - k\,x$.
You are to apply various forces (weights mg) to the springs and measure the resulting stretch values x. Then analyze the data to determine the spring constant k for each spring.

DO NOT apply more weight than will double the length of any spring- this will damage the spring and render them useless for this experiment.

(2) When a spring is suspended from a support, a mass is attached to the lower end of the spring and the system is allowed to come to rest. The mass is then at equilibrium under the force of gravity mg and the upward force of the spring $k\,x_0$. By *eqn.* (1), $mg = k\,x_0$. If the spring is stretched an additional length x to give it a total length $(x_0 + x)$ the total spring force will be $(F + mg)$ with F representing the additional spring force required for the additional stretch x. Hooke's law gives us;

$$|F| + mg = k\,(\,x + x_0) = \,k\,x_0 + \,kx \qquad (4)$$

Because of the equality, $mg = k\,x_0$ *eqn. (4)* becomes $|F\,| = k\,x$. Here F is the additional force and x is the extension (elongation) of the spring. Forces have direction but we are interested in the magnitude here. The purpose of this discussion is to point out that if we measure the extension x from the equilibrium position established by the weight mg, we may leave out the weight, as the force acting on the spring, since this is always balanced by the initial spring force $k\,x_0$. If the mass is displaced a bit, vertically, upward or downward, and then released, it will oscillate with simple harmonic motion (SHM) having period T given by;

$$T = 2\pi\sqrt{\frac{M}{k}} \qquad (5)$$

In this formula, M must be the total mass that is oscillating with the same amplitude as the mass m that is attached to the spring. The spring itself has a mass, but only the end of the spring attached to the mass is oscillating with the same amplitude as the attached mass. The other end of the spring is not moving at all. We must add the effective mass of the spring to the attached mass to get the total mass M in eqn. *(5)*. The effective mass of the spring is 1/3 of its actual mass. This is solved via consideration of kinetic energy. If one assumes that the amount of stretch, and consequently the amount of motion (velocity) of a point on the spring that is a distance y from the suspension point is proportional to y, one may show that the total kinetic energy (KE) of the mass and spring, when the mass has velocity v is;

$$KE = \frac{1}{2}\left(m + \frac{1}{3}\,m_s\right)v^2 \qquad (6)$$

where m_s is the spring mass. Hence, $M = m + m_s/3$.

The Experiment has 4 parts:

(1) Static measurement for the spring constant k of each spring. Write down your measured results and draw a diagram.

(2) Measurement of the period T for both springs use eqn. *(5)*. See above.

Plan measurements of the periods of the motion and carry them out to obtain values of k (call them k_1 and k_2) for both of your springs. You are to measure k for them separately with the same mass m. Compare these values with the static measurements you took in part (1).

(3) Figure 2, below shows a single mass acted on by 2 springs. This is called the “parallel” arrangement because of the electrical analogue of this arrangement is a parallel combination of 2 capacitors. The effective “k” is given by,

$$k_{parallel} = k_1 + k_2 \,. \qquad (7)$$

Carry out measurements of the period T for the series system and determine by use of *eqn. (5)* the value of $k_{parallel}$. Compare this with our calculated result.

Figure 2.

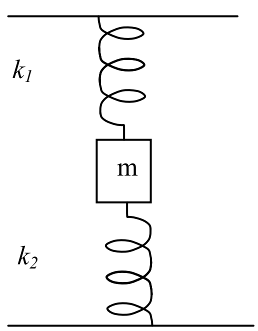
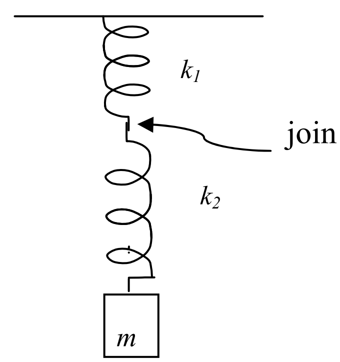


Figure 3.



(4) Figure 3, shows a single mass hanging from 2 springs. One spring is attached to a support, a second spring is attached to the bottom of the first spring. The mass m is attached to the free end of the second spring. This is called the “series” arrangement because of the electrical analogue. The effective “k” for this combination is given by,

$$k_{series} = \frac{k_1 k_2}{k_1 + k_2} \qquad (8)$$

Carry out measurements of the period T for the series arrangement and determine, using *eqn.*(5), the value of k_{series} and compare it to the calculated value in *eqn. (8)* above.

<u>Your lab report should include all tabulated results, diagrams and graphs where appropriate.</u>

Verify Hook's Law

Static Method:

1. Make a table for each one of your springs. Make columns for mass (m), position (x), weight (w), spring force (F_s), and work done (W_s).

2. Hang a 100g mass from one of the springs and measure spring displacement. Record this value and the calculated value of the spring force in your table. Do this for 5 different masses.

3. Make a plot of F_s vs. x in graphical analysis or excel, and sketch what you see in your lab report.

4. Find the slope of the graph. What is this value?

5. Find work done, and finish filling in your table.

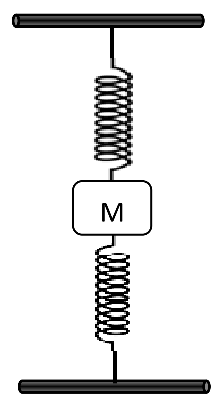
Do this for each spring.

Dynamic method:

1. Linearize the equation: $T = 2\pi\sqrt{\frac{M}{k}}$ with T as the dependent variable and M as the independent variable. What is the slope in your linear equation?

Set up your springs this way:

a) **0ÁÒÁÌÌÅÌ:** $\boldsymbol{k_{eq} = k_1 + k_2}$



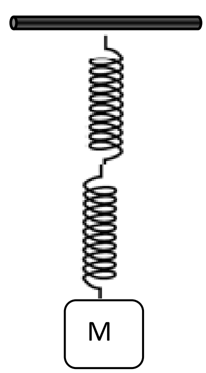
2. Make a 3x10 table for mass (m), period (T), and square of period (T^2) for 5 different masses. Record the values of masses. Measure T for each one (let it run for 10 seconds and use the average T).

3. Make a plot of m vs. T^2 in graphical analysis or excel.

4. Find k using the slope.

Repeat steps 2, 3, and 4 for this set-up.

a) 3ÅÒÉÅÓ: $\boldsymbol{\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}}$



Now you have 2 equations with 2 unknowns.

- Solve for k_1 and k_2.
- Find % different between these values and the k values from the static method.