Physical Pendulum

Introduction:

This experiment has two parts.
In part one you use 1 adjustable brass mass and a thin rod as a simple pendulum. The goal is to measure and calculate the period, as well as take data for large angles.
In part two you use 2 adjustable brass masses, one above and one below the axis of rotation, and one thin rod. Again you will measure and calculate the period of this pendulum.

Part I: Single mass Pendulum

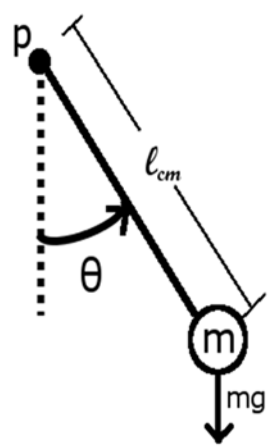
$$I_p\alpha = \sum\tau \tag{1}$$

I_p is the moment of inertia about the pivot point, a constant value. α is the angular acceleration and τ are the torques on our system.

Remember that the angular acceleration is given by

$$\alpha = \frac{d^2\theta}{dt^2} \tag{2}$$

We will start with the following pendulum,



Equation (1) can be written as

$$I_p \alpha = -(l_{cm}) \times (mg). \tag{3}$$

By using (2) and simplifying the cross product this becomes

$$\frac{d^2\theta}{dt^2} = -\left(\frac{mgl_{cm}}{I_p}\right)\sin\theta, \tag{4}$$

This is our equation of motion for the pendulum with moment of inertia I_p. Now let's get it to make a bit more sense, first call the term on the right side in parenthesis ω_0^2 so that we have

$$\sqrt{\left(\frac{I_p}{mgl_{cm}}\right)} = \omega_0. \tag{5}$$

Also assume θ is small so that we can use the approximation $\sin\theta \approx \theta$.
This leaves us with our new equation of motion

$$\frac{d^2\theta}{dt^2} = -\omega_0^2\theta, \tag{6}$$

which can be easily solved to get the equation for *theta* as a function of time as

$$\theta = \theta_0 \sin(\omega_0 t + \phi). \tag{7}$$

This is an equation for simple harmonic motion, with a frequency ω_0.*(Also an amplitude* θ_0 *and phase* ϕ*)*
Here we are interested in the period of our pendulum, so from the frequency we have

$$T_{calculated} = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\left(\frac{\mathrm{l}_1 + \frac{1}{12}\frac{\mathrm{m}_{\mathrm{ROD}} l_{ROD}^2}{\mathrm{ml}_1}}{g}\right)} \tag{8}$$

Note that this is equivalent to the two mass case (eqn. 11) with l_2 taken as zero.

Part 1: Procedure
The goal for the first part of the experiment is to measure the period of a pendulum.

First begin by insuring the rotary motion sensor is plugged into the dig/sonic 1 port. Then open the Physical Pendulum lab file.

In order to get the logger pro to start reading from the rotary motion sensor unplug the cable from dig/sonic 1 and plug it into dig/sonic II. You should now see an angle being read, move the pendulum to insure that it is reading.

Now that the system is reading, stop the motion of the pendulum and then zero out the system. **NOTE: Whenever you press collect the system will rezero to where the pendulum currently is, so make sure the pendulum is at zero, hit collect, then move the pendulum to starting point.**

Experiment part I

$m_{rod}(kg)$	$m(kg)$	l_1	θ (*deg*)	$T_{calculated}$ *(eqn. 8)*	$T_{measured}$	$\%error$
.02768						
.02768						

1. Put the brass mass on one end of the rod, near the end as shown. Measure the distance from the center of the pendulum to the edge of the mass, then add half of the height of the mass to this to get l_1.

2. Measure the period for an angle θ less than 15 degrees.
To measure the period take a set of data then highlight ten full oscillations. Take that total time for 10 oscillations, then divide that time by 10 in order to get the time per one oscillation (a.k.a the period).

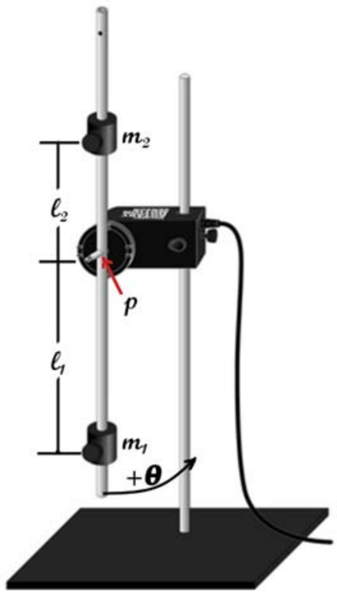
3. Calculate the period you expect from the small angle approximation, equation 8. Find the $\%$ error to your experimental value.

4. Repeat 2 and 3 for an angle θ larger than 50 degrees. See the pictures below for examples of how large the angles should be. (10 degrees on the left, 60 degrees on the right)

Question 1:
Does the small angle approximation hold for this larger value of θ?

Part II: Two masses

Now we will add a mass to the other side of the pendulum. We now have



The formula for the period is the same as part one, given by

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\left(\frac{I_p}{mgl_{cm}}\right)} \tag{9}$$

The moment of inertia is similar to part one but the center of mass has moved, so our new formula is

$$\frac{1}{12}m_{rod}L^2 + m(l_1^2 + l_2^2). \tag{10}$$

This leaves us with the equation for the period as

$$T_{calculated} = 2\pi\sqrt{\frac{\frac{1}{12}\frac{m_{rod}}{m}l_{rod}^2 + (l_1^2 + l_2^2)}{g(l_1 - l_2)}} \tag{11}$$

Question 2:

How do you expect the period to behave as l_2 approaches l_1? Explain your answer.

Experiment Part II

$m_{rod}(kg)$	$m(kg)$	l_1	l_2	$T_{calculated}$ (eqn. 11)	$T_{measured}$	$\%error$
.02768						
.02768						
.02768						

1. The distance to the first mass l_1 should be the same as in the first part of the experiment, fill this into your table.

2. Put the second mass on the other side of the pendulum as close to the center as you can. Measure l_2 the same way as l_1, measure from the center to the inner edge of the mass and then add half the height of the mass to get l_2.

3. Calculate the period using the above formula for the two mass system.

4. Experimentally measure the period using the same method as before, then calculate your percent error for the system.

5. Move the second mass about half way out on the rod and then repeat parts 2-4.

6. Move the second mass further out toward the end of the rod, but so that l_2 is still less than l_1. Repeat steps 2-4.

Question 3:
What is happening to the period of your system as l_2 is increased? Does this agree with your answer from question 2?

$$
\begin{aligned}
m_{rod} &= 27.68g, (26.24g\_without\_screw) \\
m_{weight} &= \{75.52g, 75.48g, 75.39g\} \\
l_{rod} &= 38.0cm \\
l_{mass} &= \{20.05mm, 20.06mm, 20.08mm\}
\end{aligned}
$$