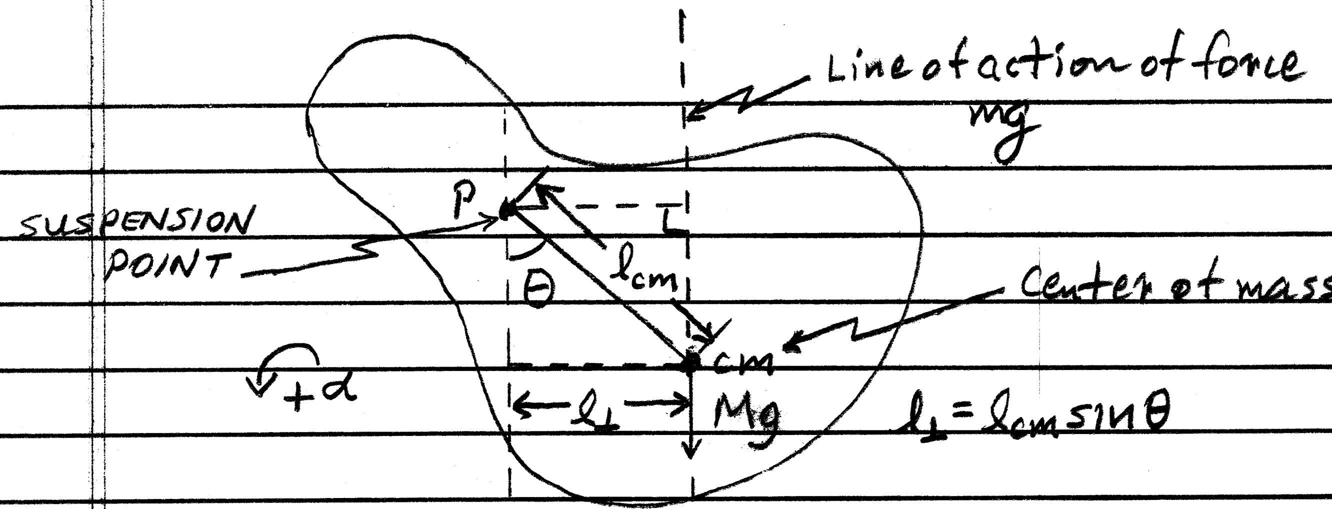
PHYSICAL PENDULUM



Angular Form of 2nd Law

$$I_p\alpha = \tau_{net} = -Mg\,l_\perp = -Mg\,l_{cm}\sin\theta$$

(minus sign since torque is CW, pos. torque is CCW.)

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

I_p = moment of inertia about pivot point P.

$$\Rightarrow I_p\frac{d^2\theta}{dt^2} = -Mg\,l_{cm}\sin\theta$$

$$\frac{d^2\theta}{dt^2} + \left(\frac{Mg\,l_{cm}}{I_p}\right)\sin\theta = 0$$

defining $\omega_0^2 = \left(\frac{Mg\,l_{cm}}{I_p}\right)$

$$\frac{d^2\theta}{dt^2} + \omega_0^2\sin\theta = 0$$

for $\theta \le 15^\circ$, $\sin\theta \cong \theta$ (in radians) to within 1.15%

$$\Rightarrow \frac{d^2\theta}{dt^2} + \omega_0^2\,\theta = 0$$

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solution $\theta = \theta_0 \sin(\omega_0 t + \delta)$

PERIOD $T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{I_p}{M g l_{cm}}}$

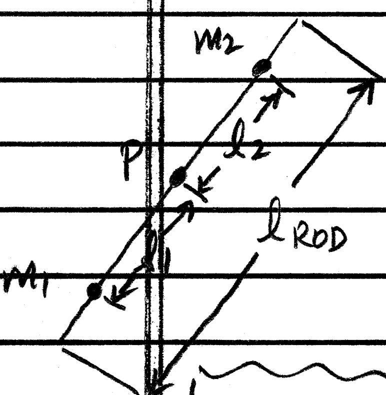
2 cases - 1 adjustable mass + thin rod
2 adjustable masses + thin rod

FOR thin rod suspended thru the center hole

$I_{ROD} = \frac{1}{12} m_{ROD} l_{ROD}^2$

$I_p = [m_1 l_1^2 + m_2 l_2^2 + \frac{1}{12} m_{ROD} l_{ROD}^2]$

WITH POINT P AS ORIGIN



$$l_{cm} = \frac{(m_1 l_1 - m_2 l_2 + m_{ROD}(0))}{(m_1 + m_2 + m_{ROD})}$$

$$l_{cm} = \frac{(m_1 l_1 - m_2 l_2)}{(m_1 + m_2 + m_{ROD})} = \frac{(m_1 l_1 - m_2 l_2)}{M}$$

$$\boxed{T = 2\pi\sqrt{\frac{(m_1 l_1^2 + m_2 l_2^2 + \frac{1}{12} m_{ROD} l_{ROD}^2)}{g(m_1 l_1 - m_2 l_2)}}}$$

Part 1 single mass $m_1 = m$, $m_2 = 0$

$$T = 2\pi\sqrt{\frac{[l_1 + \frac{1}{12} m_{ROD} l_{ROD}^2 / (m l_1)]}{g}}$$

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Part 2 $m_1 = m_2 = m$ - 2 masses

$$\boxed{T = 2\pi \sqrt{\frac{\left[l_1^2 + l_2^2 + \frac{1}{12}\frac{m_{ROD}\, l_{ROD}^2}{m}\right]}{g\,(l_1 - l_2)}}}$$

note $T \to \infty$ as $l_1 \to l_2$

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