Lab 1

Graphical Analysis

Introduction

In this first lab for Physics 211 you won't be doing any physics. This statement may have just brightened your day, however, the ideas that you will be covering today are ones that students typically find difficult to master. This lab will take them a step at a time and hopefully you will leave today feeling Physics 211 Lab is not the worst thing in the world.

Analyzing A Graph

Sometimes experiments are performed to check the validity of an established theory for which an equation is already known. One way to accomplish this is to determine one of the *constants* in the known equation. This task can be simplified by using *graphical analysis*. This lab will go through a 4-Step process to explain this procedure. For each Step an example with fake data will be discussed first and then you will work through the same procedure using your own data.

Step 1 – Linearizing An Equation

Graphical analysis will allow us to determine a *constant* by finding the slope of the line in a graph. Sometimes the plot will be linear and it is easy to find the slope. Other times the plot will be a curve and we cannot find a slope. So, the first step in this process is making sure that the plot is a straight line, i.e. linearizing your equation.

To explain this idea, we are going to use *two examples* based on the "equation of state" for an ideal gas …

$$PV = RT$$

P is the pressure (in atmospheres, atm)

V is the volume (in liters, ℓ)

T is the temperature (in Kelvin, K)

R is the ideal gas constant (in L-atm/K)

At the end of the 4-Step process we will determine the ideal gas constant, R, and see how it compares to the established value of 0.0820 ℓ-atm/K.

In the first example, let's say you are going to perform an experiment in which we vary the *temperature* and then measure the *pressure* while holding the *volume* constant. This means that the *independent* variable (the value we control) is the temperature T. The *dependent* variable would then be the pressure P. Both R and V are constants. We would then run the experiment and collect data for T and P. Before plotting the data, we must make sure that we have a linear relationship between the independent and dependent variables. In other words, we need to see if the equation is in linear form: $y = mx + b$.

The y part of the linear form must involve the dependent variable and the x part must involve the independent variable.

Linear Form	Must Involve
y part	dependent variable
x part	independent variable

We will need to rearrange or "massage" the equation so that y matches up with P and x matches up with T. Let's start by dividing both sides of the equation by V. This will leave P by itself on the left side, just like y is by itself in the linear form. See **Figure 1.** The RT/V, however, doesn't quite match with the mx part and there is nothing to match up with b. So, we rearrange the fraction a little as well as adding a **0**. **See Figure 1b.** Now the two equations match. The y (y-axis) is equivalent to the P, the m (the slope) is equivalent to the R/V, the x (x-axis) is equivalent to the T, and the b (the y-intercept) is equivalent to the **0**.

a)
$$P = \frac{RT}{V}$$
$$\downarrow \quad \downarrow$$
$$y = mx + b$$

b)
$$P = \left(\frac{R}{V}\right)T + 0$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$y = (m)x + b$$

Figure 1

In this *second example*, we are going to use the same equation but now we will change the experiment and vary the *pressure* and then measure the *volume* while holding the *temperature* constant at 350 K. This means that the *independent* variable is the pressure P. The *dependent* variable would then be the volume V. Both R and T are constants. We'd then run the experiment and collect data for P and V.

We need to go back to the original equation so that it can be massaged into linear form. In this example, the y part needs to involve V since it is the dependent variable and the x needs to involve P since it is the independent variable. **Figure 2** shows the algebraic procedure for this. The process is similar to what we did before but it is different in how we rearranged the mx part. Now, the y (the y-axis) is equivalent to the V, the m (the slope) is equivalent to the RT, the x (the x-axis) is equivalent to the $1/P$, and the b (the y-intercept) is equivalent to the **0**.

a)
$$V = \frac{RT}{P}$$
↓ ↓
$$y = mx + b$$

b)
$$P = (RT)\frac{1}{P} + 0$$
↓ ↓ ↓ ↓
$$y = (m)\ x + b$$

Figure 2

Exercise #1

Before moving on to Step 2 in the process you are going to apply what you have learned so far. In each of the problems below, a theoretical equation is given and the variables and constants are identified. In some cases you are told which variable is to be the independent one and which is to be the dependent one. In other cases it is up to you to decide. Do the following for each of the given equations ….

   I.   "massage" the equation into linear form

   II.   identify in the linearized equation your y, m, x, and b

EXAMPLE

The impedance of a series RC circuit is given by:

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega^2 C^2}\right)}$$

Z is the dependent variable and ω is the independent variable. R and C are constants.

I.

1. $Z = \sqrt{R^2 + \left(\frac{1}{\omega^2 C^2}\right)}$

2. $Z^2 = R^2 + \left(\frac{1}{\omega^2 C^2}\right)$

3. $Z^2 = \left(\frac{1}{C^2}\right)\left(\frac{1}{\omega^2}\right) + R^2$

II.

$$Z^2 = \left(\frac{1}{C^2}\right)\left(\frac{1}{\omega^2}\right) + R^2$$
↓ ↓ ↓ ↓
$$y\ =\ m\quad x\ +\ b$$

Do the following on a separate sheet of paper …

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1) The resonant frequency, ω, of a parallel L-C circuit is given by:

$$\omega = \frac{1}{\sqrt{LC}}$$

ω and C are measured variables, where ω is the independent variable, C is the dependent variable, and L is held constant.

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2) The linear expansion of a solid, ℓ, is described by:

$$\ell = \ell_0(1 + \alpha\Delta T)$$

ℓ and ΔT are measured variables. ℓ_0 and α are constants.

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3) A conical pendulum has a period, T, given by:

$$T = 2\pi\sqrt{\frac{l(cos\theta)}{g}}$$

T and θ are measured variables, where θ is the independent variable. l and g are constants.

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4) The wavelength, λ, of the light in the Balmer series of the spectrum of the hydrogen atom is given by:

$$\frac{1}{\lambda} = B\left(\frac{1}{4} - \frac{1}{n^2}\right)$$

λ and n are measured variables, where n is the independent variable. B is a constant known as the Rydberg constant.

Step 2 – Determining Axes and Modifying Data Table

We are now going to go back to our *examples* from Step 1. We massaged our equation into linear form and now we need to determine how to plot our data, in other words, we need to determine what to plot on the x-axis and the y-axis. By looking at the linearization results, we will see how to do this.

Let's go back and look at the results of the *first example* we did. See **Figure 1.** We determined that on the y-axis we are going to graph the pressure data. On the x-axis we are going to graph the temperature data. So, we would make a graph of *P* vs. *T*. The plot would be linear and we can calculate a slope.

In the *second example*, we determined that on the y-axis we are going to graph the volume data and on the x-axis we are going to graph "1 divided by the pressure data", *1/P*. See **Figure 2.** If we tried to graph the data as simply *V* vs. *P*, the plot would come out as a curve (i.e. non-linear) and we would not be able to calculate a slope.

To show that let's say we took data based on the *second example* we've been discussing, the one in which we varied the *pressure* and then measured the *volume* while holding the *temperature* constant. Plotting that data directly gives us a nonlinear graph as shown.
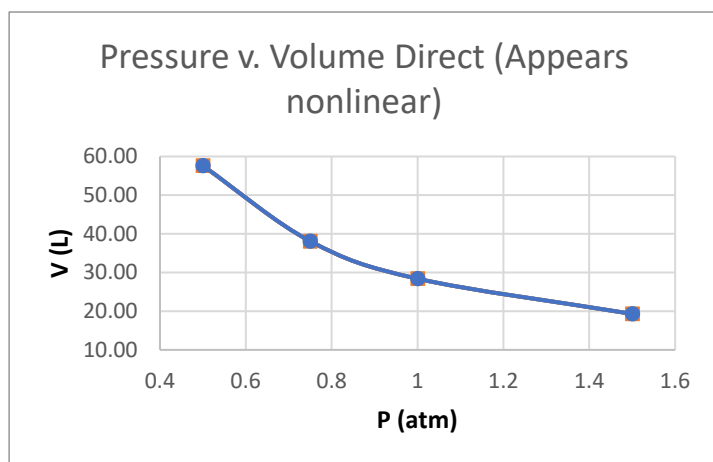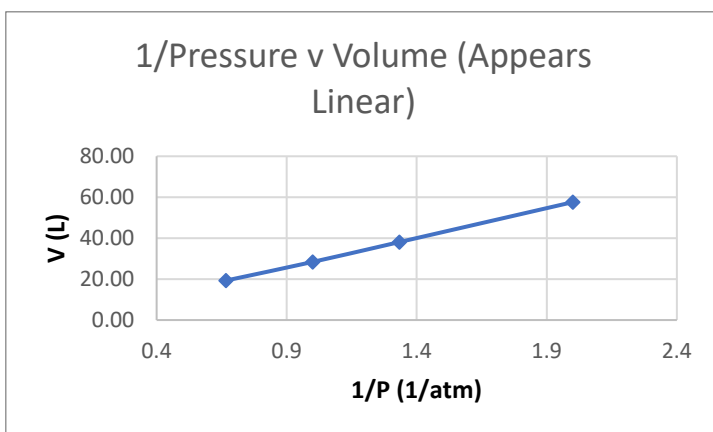

Variables	
P (atm)	V (L)
0.50	57.6
0.75	38.1
1.00	28.4
1.50	19.3

Figure 3

So, in order to get a straight line we must make a graph *V* vs. *1/P*. This process is also known as "forcing a line" in order to "force" the curved line to become a straight line.

We need to make another column with a title of *1/P*. We would then take each of the pressure data and divide them into 1, putting these new values into an additional column. This will allow us to make a graph of *V* vs. *1/P* which will give us a linear plot as shown below.

1/Pressure v Volume (Appears Linear)

Variables		
P (atm)	1/P (1/atm)	V (L)
0.50	**2.00**	57.6
0.75	**1.33**	38.1
1.00	**1.00**	28.4
1.50	**0.67**	19.3

In this second *example* only the x-axis needed modification, but there are equations in which you will need to modify BOTH of your axes in order to get a linear plot.

Exercise #2

You are now going to do the same process described above but you will be using the equation and results from **Problem #4** in **Exercise #1.**   For your data you will use the data in Figure 4. Do the following …

Variables		
n	λ (m)	
3	6.56×10^{-7}	
4	4.86×10^{-7}	
5	4.34×10^{-7}	
6	4.10×10^{-7}	
7	3.97×10^{-7}	

Figure 4

1) Based on your results from **Problem #4** in **Exercise #1**, state how you are going to graph your data on the x-axis and the y-axis.

2) Based on your answer to the above, modify your data table (like in the *example* in **Figure 3b)** so that when you make your graph you will get a straight line. Do not leave your data in fraction form. Use a calculator to get a decimal number.

Step 3 – Plotting Your Data

Computers are very handy tools. Sometimes, however, they make things too easy for students and they never learn some basic skills. For example, many students don't have a clue how to properly graph by hand. There are two main ideas to keep in mind while plotting data. They are …

   I.   **Use more than half of the graph paper.**

   II.   **Make your data easy to plot.**

In order to make a proper graph we need to strike a balance between these ideas. To demonstrate this we are going to go through an *example* using the *fake data* in **Figure 5**

| X	Y
5	920
12	1130
20	1610
32	2180
44	2730
51	3300
63	3960

Figure 5

Before actually plotting the fake data in this *example* we need to assign values on the horizontal and vertical axes. Let's examine the data. The X data has a minimum of about 0 and a maximum of about 70. Therefore, on one axis we will spread out the assigned axis values from 0 to 70. The Y data has a maximum of about 4000, however, we wouldn't want to assign the axis values from 0 to 4000. Since there is no Y data from 0 to 920, we wouldn't be using that region of the graph paper. We should then only assign values from roughly 900 to 4000.

Next, we need to decide on the orientation of the graph paper. The graph paper used in this lab has 18 by 25 major divisions each with 10 subdivisions. (Examine the graph paper at your table.) The X data gives a spread of 70. The Y data gives a spread of 3100 … (4000 – 900 = 3100). Typically, one chooses to put the data that has the largest spread on the axis that has the most divisions. So, in this case, the   Y axis will have 25 major divisions and the X axis will have 18 major divisions.

Ok, now we are going to assign the values for the divisions. Let's discuss some options for the X data.

Option #1: we could take the 18 major divisions and divide that into the "spread" of 70 [70 / 18 = 3.89]. So, each major division would have values that are multiples of 3.89 and the subdivisions would have

values of 0.389. **See Figure 6.** This certainly would make the graph very large but it would also make the data very difficult to plot. So, let's go to Option #2 …

Figure 6

3.89   7.78   11.7
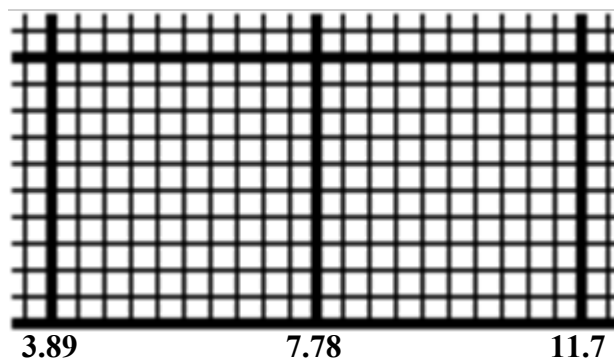
Option #2: We can make the data extremely easy to plot if we make each major division equal to 10 with subdivisions having a value of 1. **See Figure 7.** However, in examining the graph paper, we would be using less than half of the graph paper on the x-axis.

Figure 7

40   50   60   70
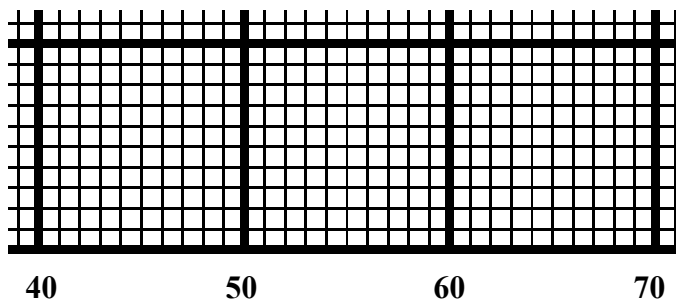
Option #3: The best option is to expand the axis by doubling the number of subdivisions in between the assigned values of 10. **See Figure 8.** This will accomplish two things. First, we will be using more than half of the graph paper (about 3/4 of it). Second, the data will still be fairly easy to plot with each major division having a value of 5. Each subdivision will then have a value of a half or, a better way to think of it, every two subdivisions will have a value of one.

Figure 8

40   45   50   55
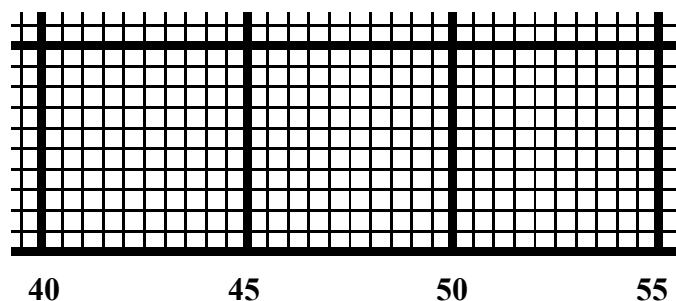
In examining the Y data, we see that the numbers are quite large and simply assigning a value of 10 to the major divisions won't work. If we try multiplying by a factor of 10, making each major division worth 100, we won't have enough room with a spread of 3100 and only 25 major divisions. Since we now have the opposite problem as before, we can use the opposite solution. Instead of expanding the number of subdivisions we can contract them and as a result each major division is now worth 200. The subdivisions will then have a value of 20. **See Figure 9.**


Figure 9
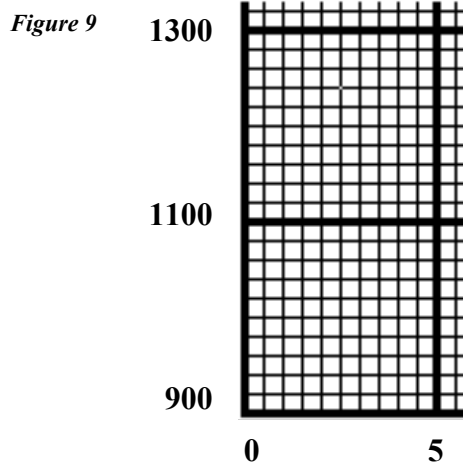
Exercise #3

You are now going to do the same process as in Step 3 but you will be using the data you calculated in *your* modified data table from **Exercise #2**. Determine how you are going to spread out your data for each axis and then plot it. You will be graded on how well you adhere to what was discussed in this section.

Step 4 – Finding Your Slope and Constant

We are now going to go back to our *example* from Step 2. Now that we have made our graph we can then proceed to find the slope. As an *example* let's say we have graphed the data from the modified data table from Step 2 using the data in **Figure 3**. **See Graph 1.** Notice how the axes values were chosen so that a majority of the graph is used. Also notice the labeling of each axis with a name and the units.
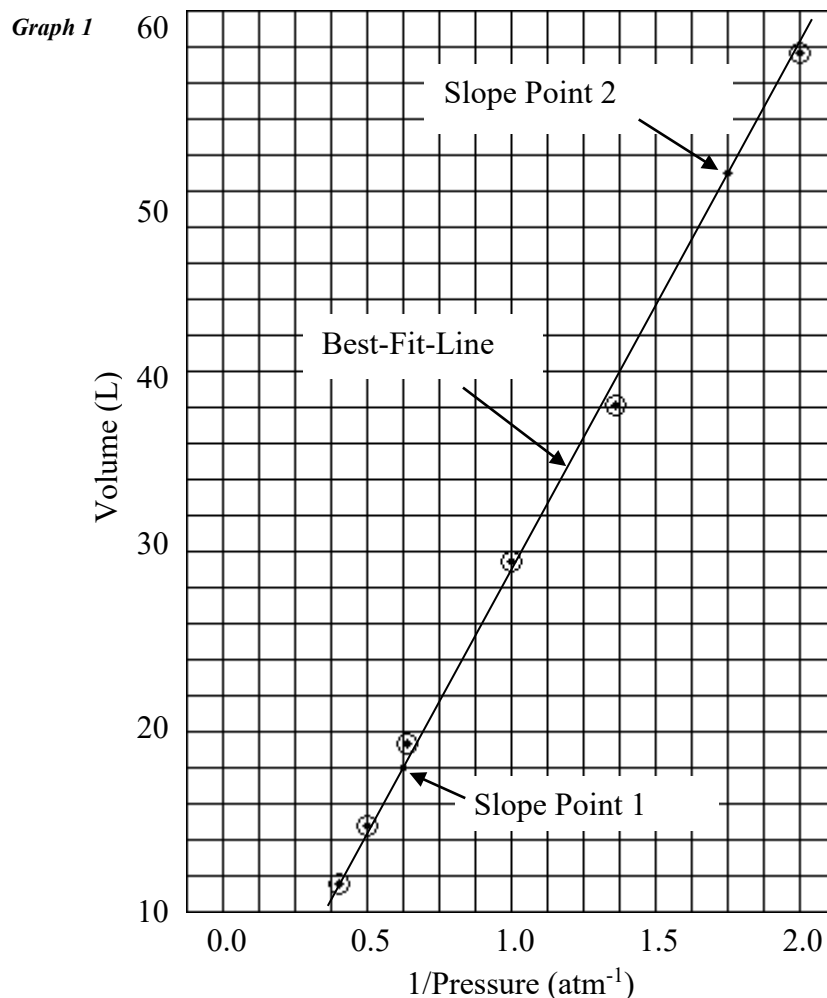
Graph 1

The line in the graph is called a "best-fit-line" (BFL). The BFL is a line that best approximates the linear nature of your data. A BFL is drawn by visually equalizing the amount of deviation of the data points above and below the line. It is NOT a requirement that your data points actually fall on your BFL as in this case. However, if a data point does fall on the BFL, it is merely coincidence and it does not make that data point "better" than any other.

Let's calculate the slope of the BFL in this example. To do this, we need to choose two points on the BFL based on the following two criteria. First, we want the points to be far apart on the line. **See Graph 1.** This will give you a more accurate slope. The second criterion is to choose the points so that they are easy to read. In **Graph 1**, two slope points are shown. Notice that the **Slope Point 1** is easy to read because it falls right at an intersection of a horizontal and a vertical line. There isn't another point at an intersection so we should at least choose a point that falls on at least one line. See **Slope Point 2 marked on Graph 1**. The coordinates of these points, as well as the slope calculation, are given in **Figure 10.**

Slope Point 1 $(0.625\ 1/atm,\ 18\ L)$ Slope Point 2 $(1.75\ 1/atm,\ 51\ L)$	$m = \dfrac{Rise}{Run} = \dfrac{\Delta V}{\Delta\left(\frac{1}{P}\right)} = \dfrac{(V_2 - V_1)}{\frac{1}{P_2} - \frac{1}{P_1}}$ $m = \dfrac{51L - 18L}{1.75\frac{1}{atm} - 0.625\frac{1}{atm}}$ $m = 29.33\dfrac{L}{\frac{1}{atm}} = 29.33\ L \cdot atm$

Figure 10

The entire purpose of this procedure has been to find and verify a constant in our equation. Now we can finally find it. The constant for the equation we've been dealing with is the ideal gas constant, **R**. The other constant, **T**, we stated earlier would be held at 350 K.

From our *second example* in Step 1, we have already determined that the slope, **m**, was determined to be equivalent to **RT**. **See Figure 2**

The calculations are done below to determine the value of **R** and also the percent error associated with its true value of 0.0820 L-atm/K. **See Figure 11**

$m = RT$ $R = \dfrac{m}{T}$ $R = \dfrac{29.33\ L \cdot atm}{350\ K}$ $R = 0.0838\ L \cdot atm/K$	$\% \ Error = \dfrac{\lvert Actual - Calculated \rvert}{Actual} \times 100$ $\%Error = \dfrac{\lvert 0.0820 - 0.0838 \rvert}{0.0820} \times 100$ $\% \ Error = 2.2\%$

Figure 11 - Determining R and % Error

Exercise #4

You are now going to do the same process as in Step 4 but you will be using the graph you made in **Exercise #3.**

Start by drawing your BFL using the clear ruler on your table.

Based on your BFL, calculate the slope of the line.

Use your slope to calculate the Rydberg constant, **B**, based on your linearized equation from **Problem #4** in **Exercise #1**.

Calculate a percent error by comparing your calculated value to the actual value of 1.097×10^7 m^{-1}.

Exercise #5

Now you are going to do the entire 4-Step process over again using data that you will collect yourself. At your lab table you will find a ball hanging from a string of length, **L**, a timer, and a meter stick. You are going to allow the ball to swing back and forth and then measure the period of this pendulum, **T**.

The equation of the period of a pendulum is given by …

$$T = 2\pi \sqrt{\frac{L}{g}}$$

T and **L** are measured variables. **g** is a constant.

In taking your data, let the ball swing back and forth 10 times. Measure that entire time and then divide this time by 10. This will give you the period for one swing. Make sure that the swing of the ball is small, only about 15˚ to each side. Take data for at least 5 different lengths that vary from 15 cm to 90 cm. Use the black metal clip to change the length of the string. Make a chart to put your data in.

Using the procedure you learned in this lab, determine the constant, **g**, whose value is 9.8 m/s². It is up to you to determine your independent and dependent variables. You should also calculate a percent error.