Physics 101

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Solutions to Problems - Introductory Chapter (Underpinnings)

1. *Which of the following measurements results in a fundamental property?*
Answer: D. measuring the time required for a pendulum to complete a full swing back and forth
The four fundamental properties are length, mass, time and electric charge. A refers to area which requires two measurements of length, B refers to density which requires a measurement of mass and three measurements of length to get volume, C refers to volume which requires three measurements of length. All of these are derived properties. The only choice which refers to a single measurement is D which is a single measurement of time - a fundamental quantity.

2. *How many inches are there in 1 mile? (Remember: 1 mile = 5280 feet)*
We are starting with 1 inch and we want to convert to miles, i.e., we want to multiply units of inches by one or more conversion factors to ultimately convert to miles:

$$1\ \cancel{\text{mile}} \times \frac{5280\ \cancel{\text{ft}}}{1\ \cancel{\text{mile}}} \times \frac{12\ \text{inch}}{1\ \cancel{\text{ft}}} = \frac{12 \times 5280}{1}\ \text{inch} = 63360\ \text{inch} = 6.336 \times 10^4\ \text{inch}$$

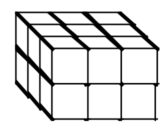
Note that the units of miles and ft cancel.

3. *How many seconds are there in one year?*

$$1\ \cancel{\text{yr}} \times \frac{365\ \cancel{\text{day}}}{1\ \cancel{\text{yr}}} \times \frac{24\ \cancel{\text{hr}}}{1\ \cancel{\text{day}}} \times \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{hr}}} \times \frac{60\ \text{sec}}{1\ \cancel{\text{min}}} = \frac{365 \times 24 \times 60 \times 60}{1}\ \text{sec} = 3.2 \times 10^7\ \text{sec}$$

4. *How many years are there in one second?*
Having previously done problem 3, we found that we have 3.7×10^7 sec/1 yr. The answer to this problem is just the recirocal of that, i.e. $1/(3.7 \times 10^7$ sec/1 yr$) = 3.1 \times 10^{-8}$ yr/1 sec. However, assuming that we had not done #3 first, we would have to start from scratch and the detailed calculation is shown below

$$1\ \cancel{sec} \times \frac{1\ \cancel{min}}{60\ \cancel{sec}} \times \frac{1\ \cancel{hr}}{60\ \cancel{min}} \times \frac{1\ \cancel{day}}{24\ \cancel{hr}} \times \frac{1\ yr}{365\ \cancel{day}} = \frac{1}{60 \times 60 \times 24 \times 365}\ yr = \frac{1}{3.2 \times 10^7}\ yr = 3.1 \times 10^{-8}\ yr$$

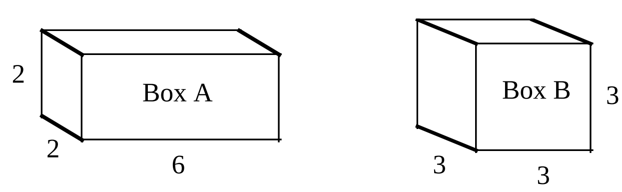


Questions 5-7 refer to the figure on the right. An object is constructed from 18 individual cubes as shown in the figure. Each individual cube measures 2 m on each edge and has density 3.6 kg/m^3.

5. *How many unit cubes are there?*
Answer: D. 144 By definition, a unit cube is a cube that measures one unit (in this case, one meter) on each edge. The object consists of 18 cubes that measure 2 m on each edge. Each one of these 2 x 2 x 2 cubes contains 8 unit cubes and, since there are 18 of the 2 x 2 x 2 cubes there are 18 x 8 = 144 unit cubes. This is what we mean when we say that the volume of the object is 144 m^3 which is what is obtained by multiplying L x W x H = 6 m x 6 m x 4 m = 144 m^3

6. *What is the density of the object?*
Answer: A. 3.6 kg/m^3 The density of each of the 2 x 2 x 2 cubes that make up our larger object is 3.6 kg/m^3. This means that the mass of any unit cube is 3.6 kg. If we stack two unit cubes together the total mass is twice as great, but the amount of mass per one unit cube is still 3.6 kg. No matter how many unit cubes are used to build a larger object, the mass per single unit cube (which is the density) does not change.

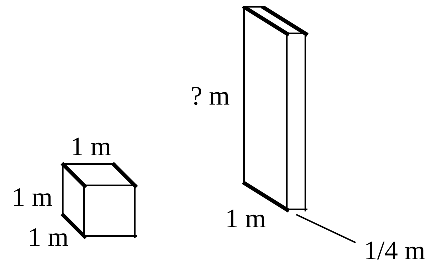
7. *What is the mass of each of the unit cubes of which the object is composed?*
Answer: C. 3.6 kg As discussed in the previous problem, the mass of one unit cube is the density. Thus, each unit cube has a mass of 3.6 kg.



8. *Consider the two boxes (with lids) shown below.*
The dimensions of the boxes are given in meters. Answer the following questions about the boxes. Explain your reasoning.
(a) Suppose these boxes, including their lids, are to be made out of the same very thin plywood. Which box requires more wood?
For box A, the total surface area is the area of all six faces which is 2(2 x 2) + 4(2 x 6) = 56 m^2. For Box B, the total surface area is 6(3 x 3) = 54 m^2. Box A has more surface area and will require more wood.
(b) Which box holds more peanuts?
The box that has the greater volume will hold more peanuts. The volume of A is 2 x 2 x 6 = 24 m^3 and, for Box B, the volume is 3 x 3 x 3 = 27 m^3. Box B will hold more peanuts.
(c) Which box is heavier (empty)?
For the empty box, the only thing that determines the mass is the wood and the box with the greater amount of surface area would have the greater mass—this is box A.

9. *An aluminum cube (on the left in the figure below) is reshaped (without changing the mass) into the figure on the right. What is the unknown dimension of the new block?*
The thing that determines density is the material of which something is made (For example, the density of aluminum is different from that of copper, but aluminum objects of all sizes and shapes have the same density). Since both blocks are made of

the same material (aluminum), they have the same density. The problem states also that both blocks have the same mass. If the density (ρ) and mass (m) is the same, then, from the density formula, it should be clear that the volume (V) of both blocks must be the same. (ρ = m/V, and if ρ and m are the same for two blocks, so must V be the same.) The volume of the original cube is 1 m^3. If we label the unknown dimension of the new block as "y", as shown in the figure, the volume of the new block is y x 1 x 1/4 m^3 = y/4 m^3. This must be equal to the volume of the original cube which is 1 m^3., i.e., y/4 = 1 and y = 4 m.



10. *A statue made out of pure iron has a volume of 3500 cm^3.*

(a) What is the mass of the statue?

Your notes give the density of iron as 7.85 g/cm^3. Thus, the mass will be M = ρ x V = (7.85 g/cm^3) x 3500 cm^3 = 27,475 g = 2.75 x 10^4 g.

(b) What would be the mass of an identical statue made from pure aluminum?

Your notes give the density of aluminum as 2.7 g/cm^3. Thus, the mass will be M = ρ x V = (2.7 g/cm^3) x 3500 cm^3 = 9450 g = 9.45 x 10^3 g.

11. *The mass of an electron is roughly 10^{-30} kg. If the mass of the proton is roughly three orders of magnitude larger than the mass of the electron, the approximate mass of the proton is*

Answer: B. 10^{-27} kg The fact that the mass of the proton is three orders of magnitude larger than the mass of the electron means that the mass of the proton is three factors of ten greater, i.e.,
mass of proton = (mass of electron) x 10 x 10 x 10 = 10^{-30} x 10^3 = 10^{-27}.

12. *Imagine that a force of magnitude 350 pounds presses down on a surface that has an area of 10 square meters. Explain <u>in words</u> what is the meaning of the ratio 350/10?*

The ratio 350/10 (which is equal to 35) means that there is a force of 35 pounds pressing down on *<u>each unit square</u>* of the surface—that is, we say that the force *<u>per unit area</u>* is 35 pounds per square inch. By the way, the amount of force pushing on a unit area is called *pressure* .

13. *How many nanometers are there in a centimeter?*

Just as we did the conversions in problems 2 and 3, we are here starting with 1 centimeter (cm) and we want to convert to nanometers (nm), i.e., we want to multiply units of cm by one or more conversion factors to ultimately convert to nm:

$$1\ \not{cm} \times \frac{1\ \not{m}}{100\ \not{cm}} \times \frac{10^9\ nm}{1\ \not{m}} = \frac{10^9}{10^2}\ nm = 10^7\ nm$$

14. *Cube A is 8 cm on a side and cube B is 3 cm on a side.*

(a) How many times larger is the cross-sectional area of cube A than the cross-sectional area of cube B?

To make a comparison, we will form the ratio. The ratio of two areas is always equal to the square of the ratio of the length dimensions, i.e.,

$$\frac{A_A}{A_B} = \left(\frac{L_A}{L_B}\right)^2 = \left(\frac{8}{3}\right)^2 = (2.67)^2 = 7.1$$

(b) How many times larger is the total surface area of cube A than the surface area of cube B?

Since there are six faces, the total surface area of a cube is 6 x the area of one face. Thus, when we form the ratio to compare the surface areas of the two cubes, we simply get a factor of 6 in numerator and denominator of the ratio which cancels out. Thus, the ratio of total surface areas is the same as the ratio of the areas of a single face.

$$\frac{\not{6}A_A}{\not{6}A_B} = \frac{A_A}{A_B} = 7.1$$

15. *The radius of a a penny is about 0.7 times the size of a quarter. How much bigger is the area of a quarter compared to that of a penny?*

To compare the areas, form the ratio, i.e., A_Q/A_P where A_Q is the area of the quarter and A_P is the area of the penny. The ratio of areas is the square of the ratio of radii, i.e.,

$$\frac{A_Q}{A_P} = \left(\frac{r_Q}{r_P}\right)^2 = \left(\frac{1}{.7}\right)^2 = (1.43)^2 = 2.04$$

16. *Sphere A has a radius of 10 m and sphere B has a radius of 1 m.*

(a) How much bigger is the surface area of sphere A compared to sphere B?

To compare the areas, form the ratio, i.e., A_A/A_B where A_A is the surface area of sphere A and A_B is the surface area of sphere B. The ratio of areas is the square of the ratio of radii, i.e.,

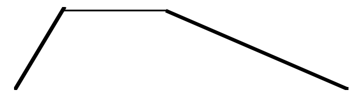
$$\frac{A_A}{A_B} = \left(\frac{r_A}{r_B}\right)^2 = \left(\frac{10}{1}\right)^2 = (10)^2 = 100$$

(b) How much bigger is the volume of sphere A compared to sphere B?

To compare thevolumes, form the ratio, i.e., V_A/V_B where V_A is the volume of sphere A and V_B is the volume of sphere B. The ratio of volumes is the cube of the ratio of radii, i.e.,

$$\frac{A_A}{A_B} = \left(\frac{r_A}{r_B}\right)^3 = \left(\frac{10}{1}\right)^3 = (10)^3 = 1000$$

17. *You are a farmer looking to buy a farm. One day you see advertised in the newspaper a 10 acre parcel (an acre is a measure of land area) of farmland shaped like the trapezoid shown in the figure for $20,000. Two days later, you see another parcel of land, priced at $28,000, in which all of the length dimensions shown in the figure to the right are 1.25 times longer. Which is a better deal, that is, for which parcel is the price per acre lower?*



If all the length dimensions of an irregular figure are uniformly increased by the same factor, the area increases by the square of that factor. In this case, we can say that the area of the larger farm is $(1.25)^2$ =1.56 times larger than the area of the smaller farm. Since the area of the smaller farm is 10 acres, then the area of the larger farm is 1.56 x 10 acres = 15.6 acres. The price per unit area of the smaller farm is $20,000/10 = $2,000 per acre. The price per unit area of the larger farm is $28,000/15.6 = $1,795 per acre and the larger farm is a better deal.

18. *The linear dimensions of a storage tank are reduced to half their former values.*

(a) By how much does the overall surface area of the tank decrease?

The surface area changes as the square of the length dimensions (i.e., the linear dimensions). Thus, if the linear dimensions are reduced by 1/2, the area is reduced by $(1/2)^2$ = 1/4.

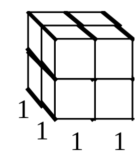
(b) By how much does the volume of the tank decrease?

Thevolume changes as the cube of the length dimensions (i.e., the linear dimensions). Thus, if the linear dimensions are reduced by 1/2, the volume is reduced by $(1/2)^3$ = 1/8.

19. *The human lungs have a relatively small volume, yet a very large internal surface area. Why is this important, and how is this accomplished?*

The oxygen in the air that we breath is absorbed into the bloodstream through the inside surface of the lung. If the lungs were smooth surfaces, the surface area-to-volume ratio would be relatively small and there would be a limited amount of air that could pass through the surface into the bloodstream. Instead of a smooth surface, the lungs have thousands of little sacs (called alveoli) which significantly increase the surface area while keeping the volume relatively unchanged. This vastly increases the surface-to-volume ratio and allows our lungs to process much more air.

20. *A cube 2 cm on a side is cut into cubes 1 cm on a side.*



(a) How many cubes result?

As shown in the figure to the right, there will be eight cubes.

(b) What was the surface area of the original cube and what is the total surface area of the eight smaller cubes?

The original cube had a total surface area of 6(2 x 2) = 24 cm^2. Each of the separated cubes has a surface area of 6(1 x 1) = 6 cm^2 and, since there are eight such cubes, the total surface area is 8 x 6 cm^2 = 48 cm^2. The area has doubled—which makes sense since there are now more exposed faces which were previously "inside" the larger cube.

(c) *What are the surface-to-volume ratios of the original cube and the combination of all the smaller cubes?*

The volume of the original cube was 2 x 2 x 2 = 8 cm^3 and the surface-to-volume ratio was 24/8 = 3. After the cube was cut up, the total volume is still 8 cm^3, but now, the surface-to-volume ratio is 48/8 = 6, which is larger than before.