Determining the Half-Life of a Radioactive Material

Neutron Activation

Neutron activation is a technique for identifying the composition of a material by making some of it radioactive then detecting the radiation emitted when it decays. When a neutron is incident on a stable nucleus there is some probability (called the cross-section) that the neutron will be absorbed by the nucleus. The resulting nucleus is usually unstable. This process of making an artificially radioactive material is called neutron activation. The nuclear reaction is similar to a chemical reaction and is written the same way. For example,

\[ n + Ag^{107} \rightarrow Ag^{108} \]  

(1)

Notice that adding a neutron is a physical change that increases the atomic mass number but does not change the chemical composition.

The unstable nuclei formed by the neutron activation decay with a characteristic half-life. As they decay γ-rays with well-defined energies and β’s are emitted. In a sophisticated experiment, the γ’s are detected, and their energies used to identify the original material. In our experiment we will count the β’s emitted, and use the half-life to identify the material.

To continue the analogy between a nuclear reaction and a chemical reaction we will include the nuclear decay in equation (1). A general mnemonic for nuclear reactions involving β’s is to remember that a neutron decays into a proton (p⁺), an electron (β⁻) and a neutrino (νₑ):

\[ n \rightarrow p^+ + \beta^- + \nu_e \]

This is effectively what happens in the nucleus during this type of nuclear decay. A specific example is the rest of equation (1):

\[ n + Ag^{107} \rightarrow Ag^{108} \rightarrow Cd^{108} + \beta^- + \nu_e \]

Note that the presence of the newly formed proton in the nucleus changes the chemistry of the atom, but does not change the atomic mass number. The half-life can be written beneath the arrow showing the decay.

Measuring \( T_{1/2} \) of the unknown

The theory of how to determine the half-life is straight-forward. The count rate follows an exponential law

\[ R(t) = R_0 e^{-\lambda t} \]
where \( R(0) \) and \( \lambda \) are respectively the initial rate and decay constant. By taking the log of both sides of the equation we find

\[
\log(R) = -t(\lambda \log(e)) + \log(R_0).
\]

Comparing this to a general equation for a straight line

\[
y = mx + b
\]

we can make the identifications:

\[
y = \log(R) \\
x = t \\
m = -\lambda \log(e) \\
b = \log(R_0)
\]

That is, a plot of \( \log(R) \) vs \( t \) will give a straight line with slope, \( m \), equal to \(-\lambda \log(e)\). Using the expression \( \lambda = \frac{\ln(2)}{T_{1/2}} \), we find the half-life

\[
T_{1/2} = -\frac{(\ln(2))(\log(e))}{m} = -\frac{\log(2)}{m}.
\]

The subtleties of the experiment arise:
(a) from the statistical nature of the decay rate and
(b) in trying to measure the decay rate, you are trying to measure something that is changing while you measure.

To reduce the effects of the statistics you should count for a longer time. If you did that, then the second effect would take over. Thus you need to count for a long enough time to get good statistics, and for a short enough time \( (\Delta t \ll T_{1/2}) \) so the rate does not change much during the counting interval.

To find the half-life of the unknown sample you will use a Geiger counter. You will take data for the decay rate as a function of time during the lab period. Then by plotting \( R \) vs. \( t \) you will find \( T_{1/2} \). Using the table of half-lives for different isotopes in the Appendix you can try to identify the unknown isotope.
Appendix

Statistical Nature of Radioactivity
There are two situations for which only statistical information is available in an experiment. In one, the sample is composed of such a large number of pieces (like atoms in a gas) that only average values of velocity, position, etc, can be calculated. In the other, the experiment is intrinsically of a statistical nature; for example, rolling a die when the initial conditions are inherently unknowable. There is still some debate about the correct interpretation of the statistical nature of quantum mechanics, but it seems to be settling into a position that quantum mechanical events are intrinsically statistical. In particular, even knowing all that can be known about the state of an unstable nucleus, one cannot predict when it will decay. What can be calculated and measured is the probability per unit time of its decaying.

Radioactive decay is an example of a wide variety of processes in nature where the rate of death $\frac{dN}{dt}$ of population $N$ is proportional to $N$:

$$\frac{dN}{dt} = -\lambda N$$

where $\lambda$ is called the decay constant and is characterized by the members of the population. Integrating leads to the exponential decay law

$$N(t) = N_o e^{-\lambda t}$$

Here $N_o$ is the number of unstable nuclei present at time $t = 0$. The half-life, $T_{1/2}$, is defined by

$$N(T_{1/2}) = \frac{1}{2} N_o$$

so

$$T_{1/2} = \frac{\ln(2)}{\lambda}$$

$N(t)$ is very difficult to measure directly, but rate of decay usually isn’t!

$$R(t) = \left| \frac{dN}{dt} \right| = \left| -\lambda N_o e^{-\lambda t} \right| \equiv R_o e^{-\lambda t}$$

In lab, we approximate $\frac{dN}{dt}$ by the detected constant $C \approx \Delta N$ over time $\Delta t$, so

$$R = \left| \frac{dN}{dt} \right| \approx \frac{\Delta N}{\Delta t} = \frac{C}{\Delta t}$$
Background Correction
Actually C needs to be corrected for spurious "background" (environmental) radiation counts B in time \( \Delta t \). Therefore

\[
R \approx \frac{C - B}{\Delta t}
\]

Collected decay counts C over time intervals \( \Delta t = 2 \text{ min} \) of a radioactive sample every 5 min (i.e. 3 min between count intervals) for a total of 13 intervals over about an hour. The background count rate B in time \( \Delta t = 2 \text{ min} \) was determined beforehand to an accuracy of better than 10%.

The plot or Chart was generated and formatted by selecting the data columns and clicking on the Chart Wizard button. The exponential fit was generated by clicking on the finished Chart and selecting Add Trendline in the Chart menu. In the Trendline format window, you can check the option to display the fit equation and select significant digits. You can select & double click on the Trendline to extend it by bumping up its Forecast range under the Options menu. Finally, you can click on the displayed equation and edit it directly to change \( y \) to \( C - B \) and \( x \) to \( t \). An \( R^2 \) value near unity (\( R^2 = 1 \)) indicates a good fit while an \( R^2 \) value near 0 indicates a poor one.
# Table of Half-lives

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Half-Life</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Al}^{28}$</td>
<td>2.3 m</td>
</tr>
<tr>
<td>$\text{Br}^{80m}$</td>
<td>4.58 h</td>
</tr>
<tr>
<td>$\text{Lu}^{176m}$</td>
<td>3.7 h</td>
</tr>
<tr>
<td>$\text{Kr}^{85m}$</td>
<td>4.4 h</td>
</tr>
<tr>
<td>$\text{Na}^{26}$</td>
<td>2.56 h</td>
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<tr>
<td>$\text{Er}^{171}$</td>
<td>7.5 h</td>
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<tr>
<td>$\text{Cs}^{134m}$</td>
<td>3.2 h</td>
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<tr>
<td>$\text{Sr}^{87m}$</td>
<td>2.8 h</td>
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<tr>
<td>$\text{Ba}^{139}$</td>
<td>1.42 h</td>
</tr>
<tr>
<td>$\text{Hf}^{180m}$</td>
<td>5.5 h</td>
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<tr>
<td>$\text{Mn}^{54}$</td>
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</tr>
<tr>
<td>$\text{Si}^{31}$</td>
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</tr>
<tr>
<td>$\text{I}^{132}$</td>
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<tr>
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<tr>
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<tr>
<td>$\text{In}^{115m}$</td>
<td>54.0 m</td>
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<tr>
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<tr>
<td>$\text{Tl}^{128}$</td>
<td>25.0 m</td>
</tr>
<tr>
<td>$\text{Br}^{80}$</td>
<td>18.0 m</td>
</tr>
<tr>
<td>$\text{Mg}^{27}$</td>
<td>9.45 m</td>
</tr>
<tr>
<td>$\text{P}^{32}$</td>
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