Purpose:

This lab will involve data and error analysis in order to investigate the inverse square law applied to optics. You will:

- Measure and confirm the inverse square law fall off of light intensity with distance from a small, point like source of light over at least two orders of magnitude in optical power (approximately 2 or 15 cm to over 2 meters in distance).
- Become familiar with making sensitive, high accuracy light power measurements over several orders of magnitude using a frequency counter technique.
- Investigate and understand the origins of experimental deviations from the inverse square law due to stray light and greatly reduce or eliminate them.

Introduction:

You should know from your lecture that intensity $I$ is given by the formula

$$ I = \frac{P}{A} $$

where $P$ is the power of the source and $A$ is the area.

In the case of a spherical source of light the area is that of a full sphere, which is given by

$$ A_{sphere} = 4\pi r^2 $$

So our intensity should be given by

$$ I = \left( \frac{P}{4\pi} \right) \frac{1}{r^2} $$

This shows the inverse square law, where the intensity is proportional to $r^{-2}$.

For the more general case when we work with some portion of the sphere we can use something called the solid angle, which is a 2-dimensional projection of the 3 dimensional object onto a sphere. Mathematically the solid angle $\Omega$ is given by

$$ \Omega = \int_{\phi_1}^{\phi_2} \int_{\theta_1}^{\theta_2} \sin \theta \, d\theta d\phi $$

Notice that this comes from the 3-dimensional volume integral with the radius constant

$$ V_{spherical} = \int dV = \int_{\phi_1}^{\phi_2} \int_{\theta_1}^{\theta_2} \int_0^r r^2 \sin \theta \, dr d\theta d\phi $$
For the cone shown above theta ranges from 0 to some value theta (the distance from the central axis) and phi goes from 0 to $2\pi$ so solving the integral above we get that our solid angle for a cone is

$$\Omega = 2\pi(1 - \cos \theta)$$

(1)

For the area projection on a sphere of radius $r$ we have

$$A_{cone} = \Omega r^2,$$

or

$$A_{cone} = 2\pi(1 - \cos \theta) r^2$$

Thus our intensity for a conical light source (like the ones you will use today) will be given as

$$I = \left(\frac{P}{\Omega}\right) \frac{1}{r^2}$$

(2)

*Note* You could measure the angle theta using the above diagram where

$$\tan(\theta) = \left(\frac{y}{x}\right)$$
Pre-Experiment Questions: Everyone must answer these before the lights are turned off to take data. Note that Q3 and Q5 must be done while taking data in the dark. Determine how to do them before starting the procedure.

**Question 1:**
With a normal incandescent bulb (like in the lamps on your desk) heat plays a large factor in the stability of the intensity given off by the bulb. Thus you would have to let the bulb warm up for some amount of time before taking any data. Do you expect this to be an issue with the LED sources? Explain your answer. (You may use the internet.)

**Question 2:**
Even with the lights off and the room as dark as possible there will be some small background Intensity of around 100Hz that is unavoidable. How will this affect your data? What fix could you apply for this source of error?

**Question 3:**
When you take data measure the background Intensity by taking a measurement with your light source covered or off. Record it here.

**Question 4:**
The light to frequency converter you are using to measure will reach a saturation point when it gets up to a certain value. This means that the device cannot read any higher than this value, so you will not be able to measure any Intensities higher. How would this affect your Intensity vs. Distance data? Sketch what you expect to see on a plot of Intensity vs distance if the device saturated.

**Question 5:**
When you start taking data find the value of your saturation point and list it here.

**Procedure:**
You will take measurements for two different light sources in order to compare them and their error.

**Step 1:**
Before we begin measurements copy the table on the right into your report. Note to take the 200cm point place a meter (or 2 meter) stick on the ruled tape at the end of the optical rail and having one person hold the meter stick and another hold the optical sensor head on it. Be sure that this data point is taken along the line of the optical rail.
Optical Inverse Square Law

**Step 2:**
First as a class you will take data for the LED sources. Do your best to minimize how much light is going to your neighbor’s stations, computer monitors and lamps must be off for the duration of taking data. Use the backlights on the DMMs and phone light or small flashlights for recording data. Refer to the **issues and pitfalls** section for possible error sources to minimize.

The main source of exponent error is stray light coming from a variety of different sources and reflections. Make sure that you cover the optical rail with black velvet when taking data using the solar path light, as it emits over a wider range of angles and produces more reflections off of surfaces than does the Pasco light source.

**Step 3:**
While you’re waiting for other groups to finish don’t forget to take measurements for the background and saturation point.

![Figure 2](image)

**If you are also doing the PASCO source continue, otherwise skip steps 4 and 5.**

**Step 4:**
Once everyone has finished taking their data for the LED the entire class will move on to the PASCO sources. Take note of the drastic increase in background light while using these sources use the velvet piece at your station to cover the back and sides of the light to minimize stray light.

Also notice the shadowing in the center of the circle in Figure 2 above which may affect your data.

**Step 5:**
When you finish just relax until the rest of the class finishes, do not use phones or anything during this time as it will affect their data. As soon as all groups are finished the lights will be turned on so you can move on to **analysis**.
Data Analysis:

A.) Plot the data as a scatter plot in Excel with no line connecting the data, and change the scale on both axis to log so it will be a log-log plot.

B.) Add a power law trend line and display the fit equation and $R^2$ values on the graph, formatting both the exponent and $R^2$ value to 4 decimal places. See the week one excel lab for graph formatting if you have forgotten.

When plotting be sure to check for intensity values that are at or above your devices saturation value (see question 1). These don’t reflect the inverse square law, so delete them.

Here is an example of properly formatted and labeled graph.
Discussion:

Inverse square laws have been extremely important in the historical development of physics. Newton’s law of universal gravitational attraction for the force $F$ between two spherical or point-like masses $m_1$ and $m_2$ separated by a distance $r$ between centers is $F = G m_1 m_2 / r^2$, where $G$ is the universal constant of gravitation. Coulomb’s law for the force between two point charges or two charged spheres with charges $q_1$ and $q_2$ at a sufficient distance apart is $F = k q_1 q_2 / r^2$, where $k$ is the Coulomb’s law constant. For the case at hand, the fall off of intensity or irradiance from a point or spherical source of light is $I = P / (4\pi r^2)$, where $P$ is the total (optical) power emitted by the spherical source. The question is, where in nature or practice can we find a spherical source of light? The sun and stars come to mind, and they are a good approximation to a spherical source of light. In fact, the optical inverse square law plays an important role in determining the distance of stars from earth. What about a light bulb or LED? One purpose of this experiment is to show that these other light sources can accurately be considered as point light sources over the distance ranges available for measurement in the lab, even though they may not radiate uniformly in all directions, as does a spherically symmetric point source.

Data Analysis Questions:

**Question 6:**
The angle for the cone of the LED is about 50 degrees, the PASCO source is about 25 degrees use this and equation (1) to calculate the solid angles for your light source(s)

**Question 7:**
Use these and your fit coefficient from your trendline to find the “power” of the light source(s) (note the units). *Hint: Compare your fit equation to equation (2)*
If you did multiple sources, which one is higher? Is this what you expected?
Correlation coefficient

The correlation coefficient or coefficient of determination is a measure of the goodness of the fit and represents the fraction of the total variance accounted for by the model. The closer the value is to 1.0000, the better the data fit the model function, in this case, a power law function. An $R^2$ value of 1.0000 indicates that the model perfectly fits the data.

You can also compare your exponent or slope to -2, the closer it is to -2 the better it represents the inverse square law.

Stray Reflections

The main source of error from the experiment is from stray reflections and light sources.

What reflections or stray lights could cause the same error with both light sources?

Were any of these increased or decreased with the PASCO light source compared to the LED light source?

What you need to turn in:

You should have 7 answered questions and one plots for each light source, as well as today’s data sets in your lab report.

Each plot should have a power law fit, log-log axis and a correlation coefficient.
Appendices:

Issues and Pitfalls:
Though it is easy to discuss a point source of light, in practice it can be difficult to obtain an accurate $1/r^2$ fall off of intensity with distance from the source.

Stray light due to reflections off of nearby objects, such as computer screens, walls, tables, nearby experimenters, etc. can cause deviations of the exponent from the value of 2 for a point light source. This is especially true with the Pasco light source, which has light coming out the top, the bottom, and both the front and back apertures. A sufficiently large piece of black velvet employed as a hood draped over the back and top of the Pasco light source suppresses most of the sources of deviation from pure inverse square behavior.

Non-uniform source intensity in the forward cone of light due to filament focusing and shadowing effects can also cause deviations in the exponent. These should be examined with a colored 3” x 5” card and their effect made negligible by making sure the photodetector is slightly off to the side of the non-uniformity in the beam (slight adjustment of the direction of the light source can be made by adjusting the mounting bracket).

The background light intensity in the room should be much less than the measured light intensity with the point source light on. Typical background intensity in the room should be less than 100 Hz.

It is important that the light source be warmed up for at least 10 minutes in order to stabilize its temperature and thus its light intensity. If the source is still warming up, the light intensity values will change. Thus it is wise to repeat intensity measurements several minutes apart to see how reproducible they are.

Similarly, the solar path light battery starts to decay if it is not fully charged. Check the intensity level at the end of the run close to the light source to see if it is the same as at the beginning of the run. (e.g. check the intensity at 10cm at beginning and end of a run to see how much the LED solar path light battery has drained.) Because of the battery draining effect, it is important to take sets of solar path light data that take no more than 20-30 minutes each. Do not combine LED solar path light data from different data taking runs.

Previous experiments:
The Figure and Table below summarize the results obtained with various combinations of light source, photodetector, and frequency counter. As can be seen from the graph and table, the data is high quality and ranges over up to slightly more than four orders of magnitude in intensity. The data was least square fit to a power law model expression of the form $I \propto A x^b$, where $I$ is the intensity (irradiance) in units of kHz and $x$ is the distance from the light source to the detector in cm. Background light levels were negligible in all experiments, typically 1-5 Hz.
The model results for $A$, $b$, and the coefficient of determination (COD), $R^2$, are presented in Table 1.

Log-log plot of light intensity vs. distance from source to detector for different light sources and light to frequency converters.

### Table 1. Summary of Results

<table>
<thead>
<tr>
<th>Curve</th>
<th>Light Source</th>
<th>Detector</th>
<th>Counter</th>
<th>Prefactor ($A$ (kHz/cm$^b$))</th>
<th>Exponent ($b$)</th>
<th>COD</th>
<th>Data points</th>
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<tr>
<td>1</td>
<td>incandescent, black RL8-W110-360 Super</td>
<td>TSL 237</td>
<td>Instek</td>
<td>111491.52</td>
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<td>0.998658</td>
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<td></td>
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<td>Wavetek</td>
<td></td>
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<td>White Solar path light</td>
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<td>Instek</td>
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<td>0.998508</td>
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<td></td>
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<td>Instek</td>
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<td></td>
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</tr>
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<td></td>
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<tr>
<td>4</td>
<td>incandescent, black velvet on optical rail not shown</td>
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</tr>
</tbody>
</table>

Average: $-1.99828$
Solar path light white LED optical head mounted in PASCO Basic Optics lens holder. Left: Back side of optical head showing solar cell for charging battery. Center: Foreground shows optical head of the white LED solar path light. Background shows empty lens holder and two black plastic retaining rings used to mount the optical head. Right: Front side of optical head showing white LED with retaining rings installed.

Solar Path white LED light source and detector mounted on optical rail.

Use of the Backlight feature of the DMM is important so that readings can be made in the dark with minimal disturbance to the background light levels and students at other lab stations. Keep light levels extremely low.

Keep behind the photodetector when making measurements to prevent reflections off of the experimenters.

Note the switch on the optical power sensor head. It is important to remember to switch this off after making measurements to avoid draining the battery.
Optical Inverse Square Law

**Equipment:** In order to be able to measure light intensity over several orders of magnitude with low cost apparatus, we use a light to frequency converter chip, the TSL 237, mounted to an optical post, combined with a digital multimeter (DMM) with frequency counter (GW Instek GDM-396) to function as a wide dynamic range optical power meter. This allows light power measurements over more than 5 orders of magnitude, from 1 Hz to 500 KHz, corresponding to light powers of approximately $10^{-11}$ Watts (10 pW, picoWatts) to $5 \times 10^{-6}$ Watts (5 nW, microWatts).

The chip is calibrated directly in intensity, with a conversion factor of $2.3 \text{ kHz}/(\text{W/cm}^2)$ at 524 nm. This assumes the small lens on the chip of radius 0.90 mm is illuminated uniformly over its cross sectional area. The typical frequency with no light present, the so-called dark frequency is 0.1 Hz at 25°C. A light source with small aperture, a sufficiently large piece of black velvet to drape as a hood over the Pasco light source to eliminate sources of stray light, an optical rail with sliding mount for the optical source and optical sensor head, a meter stick, translucent scotch tape, tinfoil, toothpick or other sharp point for making pinholes, and dark colored index cards are also needed. Photos of the apparatus follow. You may or may not have two different light sources to take data from.

Left: Inverse square law apparatus showing Pasco light source and sliding optical sensor head on optical rail. Sensor head connected to DMM used in frequency counter mode (GW Instek GDM-396). Right: Close up of optical sensor head with TSL237 chip centered on circular light output from source.

Examination of light source emission pattern for spatial non-uniformities that change with distance from the light source due to shadowing and focusing effects from filament and glass bulb. Make sure the output pattern of your Pasco light source does not have shadowing effects due to bulb misalignment within the housing, a common problem.