Purpose:

This lab is an experiment to verify Malus Law for polarized light in both a two and three polarizer system.

The basic description of Malus law is given as

\[ I = I_0 (\cos^2 \theta) \]

Where \( I \) is the transmitted intensity of a polarized wave, \( I_0 \) is the initial intensity of the polarized wave, and \( \theta \) is the angle between the polarizer and the initial wave’s polarization.

Part One: Two Polarizer System:

The purpose of this section is to verify that polarized light going through one polarizer conforms to Malus’ law. To do this we will take values of the transmitted intensity \( I \) as a function of the polarizer orientation \( \theta \).

The first thing to do is note that the measurements of theta may be systematically off by a few degrees, so we will insert a correction term, \( \delta \), into Malus’ law. Now we have to find what this is. The way to do this is to generate a theoretical fit and somehow minimize the distance between our data, and this fit by changing \( \delta \).

Now Malus’ law in the given form is a power function, which is difficult to work with, so let's change it to a linear one.

**Question 1:** Using Malus' law, with the \( \delta \) correction,

\[ I = I_0 \cos^2 (\theta + \delta), \]

and the trigonometric identity,

\[ \cos(2\alpha) = 2 \cos^2(\alpha) - 1, \]

show that we can rewrite the first in the linear form

\[ I = \frac{I_0}{2} \cos(2(\theta + \delta)) + \frac{I_0}{2}. \]

This should look similar to the equation of a line, \( y = mx + b \). So the plot of \( I \) against \( \cos(2(\theta + \delta)) \) should be a straight line. Let's take some data to check this.
A.) Start with both angles at 0 degrees and adjust the angle of the second polarizer in increments of around 10 degrees and take your and Intensity I, until you've taken a full 360 degrees of data points.

If your week one excel sheet includes polarization, open that and insert your data in the table. Then skip to question 2 on the next page, you've already done the next few steps.

B.) You should have two columns, one for in degrees and one for I in kHz. First add another column for $\cos(2(\theta + \delta))$ and insert this formula in the column.

C.) Reserve a cell at the top for your $\delta$ and use an absolute reference for it in the entire column. Set this to value zero for now.

D.) Excel's trig functions $\cos \theta$ has default for units in radians so be sure to convert using $360 \text{ deg} = 2 \pi(\text{ Rad})$, or use the $=\text{RADIANS}(\,)$ function in excel.
Data Analysis

A.) First plot your Intensity versus Angle.

B.) Next plot your intensity against the linearized X AXIS \( \cos(2(\theta + \delta)) \). You should see something that looks as follows.

![Two Polarizer](image)

**Question 2:**
What happens to the plot of I versus \( \cos(2(\theta + \delta)) \) if you adjust \( \delta \)? Comment on the observed behavior.

C.) Now our theory says this plot should be linear, so let's make a linear fit for it. To do this we could put a trendline to the data, but some systematic errors could make this inaccurate so instead we will use a function to estimate the line.

Luckily excel has the function we need, Linest( ).

Use the Linest() function in excel to get a slope and y intercept for the estimated line.

**Note:** To learn Linest() use the excel help and search for LINEST and scroll down until you find the slope and intercept guide. The following is what you would find.

When you have only one independent x-variable, you can obtain the slope and y-intercept values directly by using the following formulas:

Slope:
=INDEX(LINEST(known_y's,known_x's),1)

Y-Intercept:
=INDEX(LINEST(known_y's,known_x's),2)
A.) Finally we have a linear fit, use this $m$, $b$ and calculated $\cos(2(\theta + \delta))$ to find a theoretical intensity and add it to your plot of $I$ versus Angle.*

*Hint: $I_{\text{theory}} = m \cos(2(\theta + \delta)) + b$ where $m$ and $b$ are from the LINEST function and $\cos(2(\theta + \delta))$ has already been calculated, just reference the cell.

*To add it to your plot, right click your graph -> select data -> add, then add a series with the new $I$ values for $y$, but same angle values for $x$.

B.) Now minimize the distance between your data and the linear fit using the excel solver, this will solve for your systematic error $\delta$. Record your $\delta$ and its units in your lab report.

The solver is a minimization tool, use the following steps or refer to your week one excel lab.

C.) First you need something to minimize, in this case we want to minimize the sum of distances between your experimental intensities and theoretical intensities, columns D and E below. To do this use the $=\text{SUMXMY2(I_{theory \ column}, I_{experimental \ column})}$, see the equation in the formula bar.

i.) Now use the solver as shown to minimize this by changing your delta, this will set your delta so that the theory is as close to experimental as possible.

ii.) Be sure that you have checked min and unchecked the “make unconstrained variables non-negative” box then hit solve.
Question 3:
After you’ve solved for δ try adding multiple of 90 degrees to it. Notice that there is another solution for delta that gives a negative slope to your linear graph. Why? Explain.

Part Two: Three Polarizer System
For this system now we will have an initially un-polarized wave, which is then polarized by the first polarizer and then hits a second polarizer at an angle theta, which then hits a third polarizer set at 90 degrees from the first polarizer, as shown below.

For Malus’ law in this case we will multiply the original form by an additional adjustment for the third polarizer in the form of

\[ I = I_0 \cos^2(\theta) \cos^2(90 - \theta) \]

Again we can use some trig (See last page for derivation), which will get us to the linear form

\[ I_0 = -\frac{I_0}{16} \cos(4(\theta + \delta)) + \frac{I_0}{16} \]

Note that the only difference for the linearization is a 4 instead of a 2.

Three Pol Analysis
A.) First copy your two polarizer tab as a new tab, make sure you make a duplicate so that you don’t lose your earlier work. Once you have this new tab, delete the intensity values and simply change the titles and the fit formula (2 to a 4, see the eqn. above).

B.) To setup the three polarizers note the labels in the diagram. Start by putting the first and third polarizers into place, insuring that the first one is at zero degrees and that there is enough space for the second polarizer to go in between them.
C.) Now put the third polarizer at 90 degrees to the first, and look through them to minimize the light transmitted as shown below. This will ensure a proper 90 degrees, eliminating the equipment error.

Note that if you are off by even a degree in minimizing the transmitted light between one and three you will see an odd waveform as shown here. **If this happens you will likely have to retake the data**

![Graph](image_url)

D.) Finally put the second polarizer into place and start taking data as before by turning only the second polarizer. Again polarizers 1 and 3 are FIXED; you are turning only polarizer 2.

E.) Find your fit and delta the same way as part one, being sure to use the new formula.

**What you need to turn in is on next page**
What you need to turn in:

1.) For each section you need two graphs, one of $I$ v. $\theta$, and your linearized fit of $I$ v. $\cos(2(\theta + \delta))$.
2.) Make sure that your graphs are in the format shown on page 4, experimental y values should be a basic scatter plot, theoretical values should be a smoothed scatter plot. Right click the dots -> change series chart type.
3.) Don’t forget to record both of your final $\delta$s, after fitting the curves. Don’t forget units.

IMPORTANT: Be sure to check for correct units on your deltas, depending on where you convert to radians for the excel formulae your deltas will be in either radians or degrees.
Three Pol. Derivation.

Begin by taking Malus' law for the second and then the third polarizer to get,

\[ I = I_0 \cos^2(\theta) \cos^2(90 - \theta), \]
\[ I = I_0 \cos^2(\theta) \sin^2(\theta), \]

Now use the trig. identity

\[ \sin(2\theta) = 2 \sin(\theta) \cos(\theta) \]

Which gets us to the following term in \( \sin^2 \theta \).

\[ I_t = \frac{I_0}{2} \left( \frac{\sin(2\theta)}{2} \right)^2 = \frac{I_0}{8} \left( \sin^2(2\theta) \right). \]

Now use the identity

\[ \cos(2\alpha) = \cos^2(2\alpha) - \sin^2(2\alpha) = 1 - 2\sin^2(\alpha) \]

Or

\[ \sin^2(\alpha) = \frac{1 - \cos(2\alpha)}{2}, \]
\[ \sin^2(2\theta) = \frac{1 - \cos(4\theta)}{2}. \]

This leaves us with our final result

\[ I_t = \frac{I_0}{8} \left( \frac{1 - \cos(4\theta)}{2} \right) = \frac{I_0}{16} (1 - \cos(4\theta)). \]

If the first and third are misaligned this becomes more challenging, where we use the outer two pol

\[ I = I_0 \cos^2(\theta) \cos^2(\beta - \theta), \]

where beta is very close to 90 degrees. This must be solved using different methods since the initial sine substitution no longer applies.