

The Mass of Jupiter Student Guide

Introduction:

In this lab, you will use astronomical observations of Jupiter and its satellites to measure the mass of Jupiter. We will use the program Stellarium to simulate observations of the night sky. By watching the orbits of Jupiter's moons, we will determine their orbital properties. These can be used to calculate the mass of Jupiter.

Background material:

From your Introduction to Astronomy class, you have learned Kepler's laws of motion. In particular, the third law is presented as:

$$P^2 = a^3$$

In this formula, P is the period, or the time for the orbiting body to complete one orbit, and a is the semi-major axis of the orbit. The law holds as written for particular units based on Earth: P in Earth years and a in Astronomical Units, or A.U., which are defined such that 1 A.U. is the average distance between the Earth and the Sun.

Question 1: How would you determine the length of Jupiter's orbit using Kepler's third law? What information would you need?

Question 2: How would you determine the semi-major axis of Jupiter's orbit using Kepler's third law? What information would you need?

Kepler was able to accurately predict the motion of the planets that he observed, but Kepler's laws do not explain why this relationship holds. Later, Newton developed laws of motion and gravity that could be used to predict Kepler's laws. In particular, he proposed Newton's law of gravity, which determines the force between two bodies:

$$F = \frac{GMm}{r^2}$$

In this formula, F is the force, M and m are the masses of the two bodies, and a is the distance between the two bodies. G is the gravitational constant, and is equal to

$$G = 6.6737 \times 10^{-11} \frac{\text{m}^3}{\text{kg s}^2}$$

The value of G can be measured on Earth by very carefully measuring how massive objects influence each other. The uncertainty in our current best measurement of G is plus or minus $0.00080 \times 10^{-11} \text{ m}^3 / (\text{kg s}^2)$. Once we measure G on Earth, we can use the motion of objects in space to determine their masses.

Using Newton's law of gravity, and Newton's laws of motion, one can derive Newton's version of Kepler's third law:

$$P^2 = \frac{4\pi a^3}{G(M + m)}$$

Question 3: To use a formula, we need measurements of time, distance, and mass to be written in consistent units. If we have G in the standard metric units as above, in which units do we need to put the other variables?

G is in units of $\text{m}^3 / (\text{kg s}^2)$

P should be in units of

M should be in units of

m should be in units of

a should be in units of

You may notice that Newton's version of Kepler's third law includes the masses of the objects, where Kepler's version of the law for planetary motion does not. How can we reconcile this with Kepler's finding that planets orbiting the Sun all have the same relationship between P and a ?

We can understand this because the Sun is much larger than the mass of any planets. Our best measurement mass of the Sun is $1.9885 \times 10^{30} \text{ kg}$, with an uncertainty of plus or minus $0.00035 \times 10^{30} \text{ kg}$. The mass of Earth, for example, is $5.972 \times 10^{24} \text{ kg} = 0.000005972 \times 10^{30} \text{ kg}$. Therefore the total mass of Earth plus the Sun is not measurably different from the mass of the Sun. This is why all the planets appear to obey the same relationship: for systems where small bodies orbit much more massive bodies, we can ignore the mass of the small body and write Kepler's third law as

$$P^2 = \frac{4\pi a^3}{G(M)}$$

This is similar to Kepler's version, but requires different units (similar to Question 3).

Newton's laws of motion apply to all orbiting bodies, so we can use this relationship for other systems: planets orbiting other stars, moons orbiting other planets, and even stars orbiting the central mass of their galaxy.

Question 4: If we want to describe the orbit of Jupiter's moon Callisto using Newton's universal version of Kepler's third Law, what would the variables represent?

M would represent

m would represent *the mass of the moon Callisto*

P would represent

a would represent

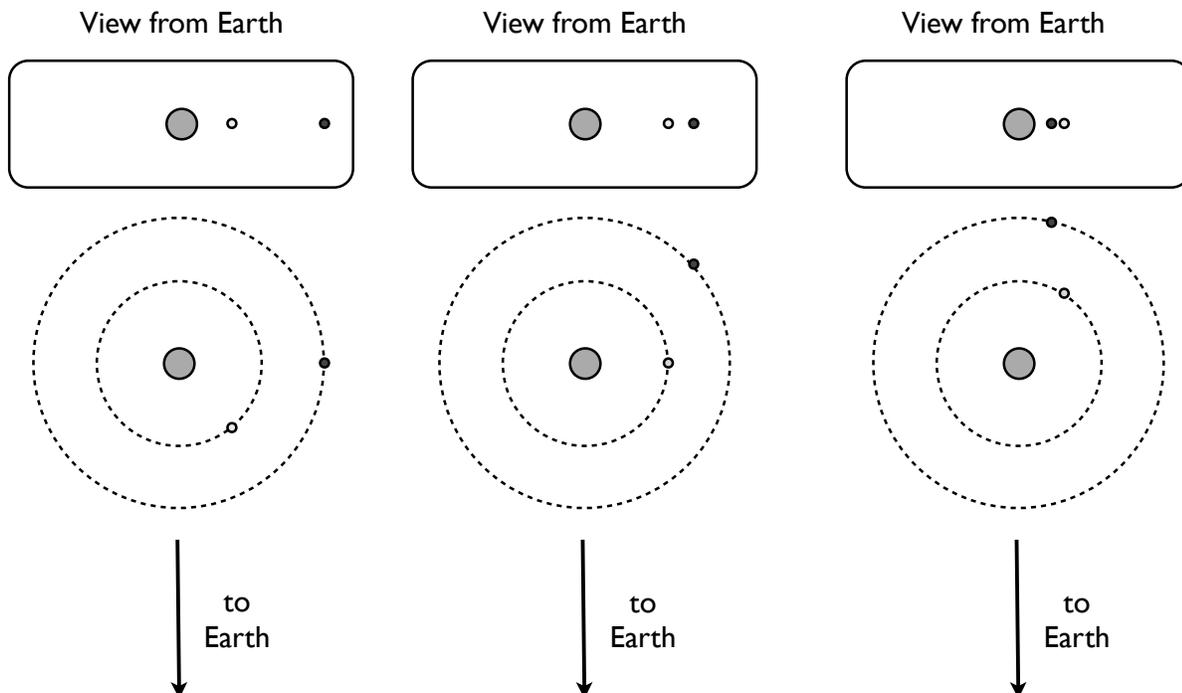
Question 5: Rearrange the version of Kepler's third law below so that the mass of the large body, M , is on the left hand side of the equation.

$$P^2 = \frac{4\pi a^3}{G(M)}$$

Question 6: We want to determine the mass of Jupiter by observing the motion of Jupiter's moon Callisto. What do we need to know to calculate the mass of Jupiter?

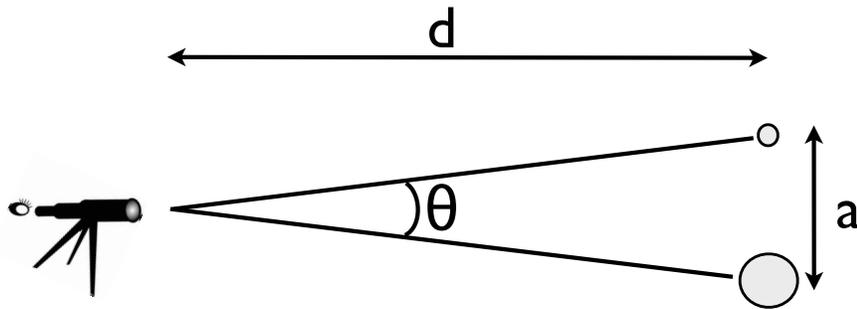
Motions of moons as seen from Earth:

The orbits of Jupiter's moons are roughly in the same plane as the orbits of the planets around the Sun. Seen from Earth, the orbital motions of Jupiter's moons make them appear to travel side-to-side:



Angular distance and true distance:

When we observe objects in the sky, we measure their size or separation in terms of the angle between them. This angle depends on the distance to the objects.



When the moons of Jupiter reach their farthest angular separation from Jupiter, they are at a distance away from Jupiter which is approximately equal to the semi-major axis of their orbit, a . If we know the distance from Earth to Jupiter, d , we can measure the maximum angular separation θ . We then can calculate a using the following formula:

$$a = d (\theta \text{ in degrees}) \frac{\pi}{180^\circ}$$

The calculated value of a will be in the same units as d , and we will need to use θ measured in degrees (as noted in the formula).

Using Stellarium:

Find the shortcut to the Stellarium program on the computer and start the program.

1. If the location in the bottom bar is not Fullerton: find Location in the left toolbar and click to open Location selection. Type Fullerton into the search bar and click on "Fullerton, United States." Click "use as default" and close the Location window. 
2. Click the play/pause button in the bottom toolbar to stop the normal flow of time in Stellarium. The time in the bottom bar should stop changing. 
3. Open the Date and Time window from the left toolbar. Change the date to 2013-10-21. Then, change the time to 00:00:00 (midnight). 
4. Find Search in the left toolbar and open Search. Type in Jupiter, and the view will scroll to center on Jupiter. You can also use the arrow keys to change the camera direction. 
5. The "page up" and "page down" keys (or the scroll button) can be used to zoom in and out on the sky. The resulting field of view (FOV) is shown in the bottom bar - this is the angular distance across the window (top to bottom) in degrees. Zoom in until the stars Castor and Pollux (the "twins" in the constellation Gemini) are visible near Jupiter, at a FOV of approximately 20 degrees.

Measuring angular distances in Stellarium:

6. Using the AngleMeasure plugin:
 - a. Enable the tool by clicking the tool-bar button, or by pressing control-A. A message will appear at the bottom of the screen to tell you that the tool is active.
 - b. Drag a line from the first point to the second point using the left mouse button. A measurement in degrees (d), arcminutes (m), and arcseconds (s) will appear.
 - c. To clear the measurement, click the right mouse button.
 - d. To deactivate the angle measure tool, press the tool-bar button again, or press control-A on the keyboard.

To test out the angular measurement tool, you will make some measurements of Jupiter and the surrounding stars.

Question 7: What is the angular separation between Castor and Pollux on 2013-10-21?

Question 8: What is the angular separation between Pollux and Jupiter on 2013-10-21?

7. The “[” and “]” keys can be used change the date forward and backward by one week, while keeping the direction of view and the time of day the same. Use these keys to advance the date by two weeks to 2013-11-4. The stars and Jupiter are now higher in the sky at Midnight.

Question 9: What is the angular separation between Castor and Pollux on 2013-11-4?

Question 10: What is the angular separation between Pollux and Jupiter on 2013-11-4?

Question 11: Did the angular separation between the stars Castor and Pollux change over the 2 weeks? Why or why not?

Question 12: Did the angular separation between the star Pollux and the planet Jupiter change over the 2 weeks? Why or why not?

8. Turn of angle measure and select Jupiter again. Switch to RA/dec mount, which will be better for tracking the motion of Jupiter through the sky. 
9. The “/” and “\” keys can be used to zoom in and out from the object of focus. Zoom in until you can see the Galilean satellites of Jupiter: Ganymede, Io, Europa, and Callisto. (One or more of these satellites may be obscured by Jupiter itself).
10. Restart the flow of time with the play/pause button. At normal speed (1 second per second), nothing much will happen.
11. Click the “fast forward” button to increase the speed of time in Stellarium. After a few clicks, you should be able to clearly see the motion of the moons over the course of the evening. When the sun rises, the sky turns blue (and a real astronomer would no longer be able to see Jupiter). When Jupiter sets below the horizon you will see the ground instead of the sky. After a couple days, press the play/pause button to halt the simulation. 

Measuring the motion of Jupiter’s moons:

In Stellarium, turn off the Ground and the Atmosphere by pressing G and A, or by clicking the icons in the text bar. Normal ground-based astronomical observations are restricted by the time of day and the orientation of the Earth, and the motions of Jupiter’s satellites must be pieced together from many evenings of observation. In this lab, we will simplify things by ignoring these effects to make periodic observations of Jupiter’s moons even during the day. 

Set the date to 2013-10-23 and the time to 20:00:00 (8PM). Make sure time is paused; you can use the Date/time window to control the time.

We will watch the inner two moons of Jupiter, Io and Europa. They move quite quickly, so we will want to record their positions every four hours in the table on the following page.

Measure the angular separation between the center of Jupiter and the center of the moon in minutes and seconds, rounded to the nearest arcsecond. If a moon is crossing Jupiter, just mark whether it is “in front” or “behind” on your sheet.

Note whether the moon is on the Western or Eastern side of Jupiter. You are looking South to see the planet move near the Ecliptic plane, so East is to the left of Jupiter, and West is to the right.

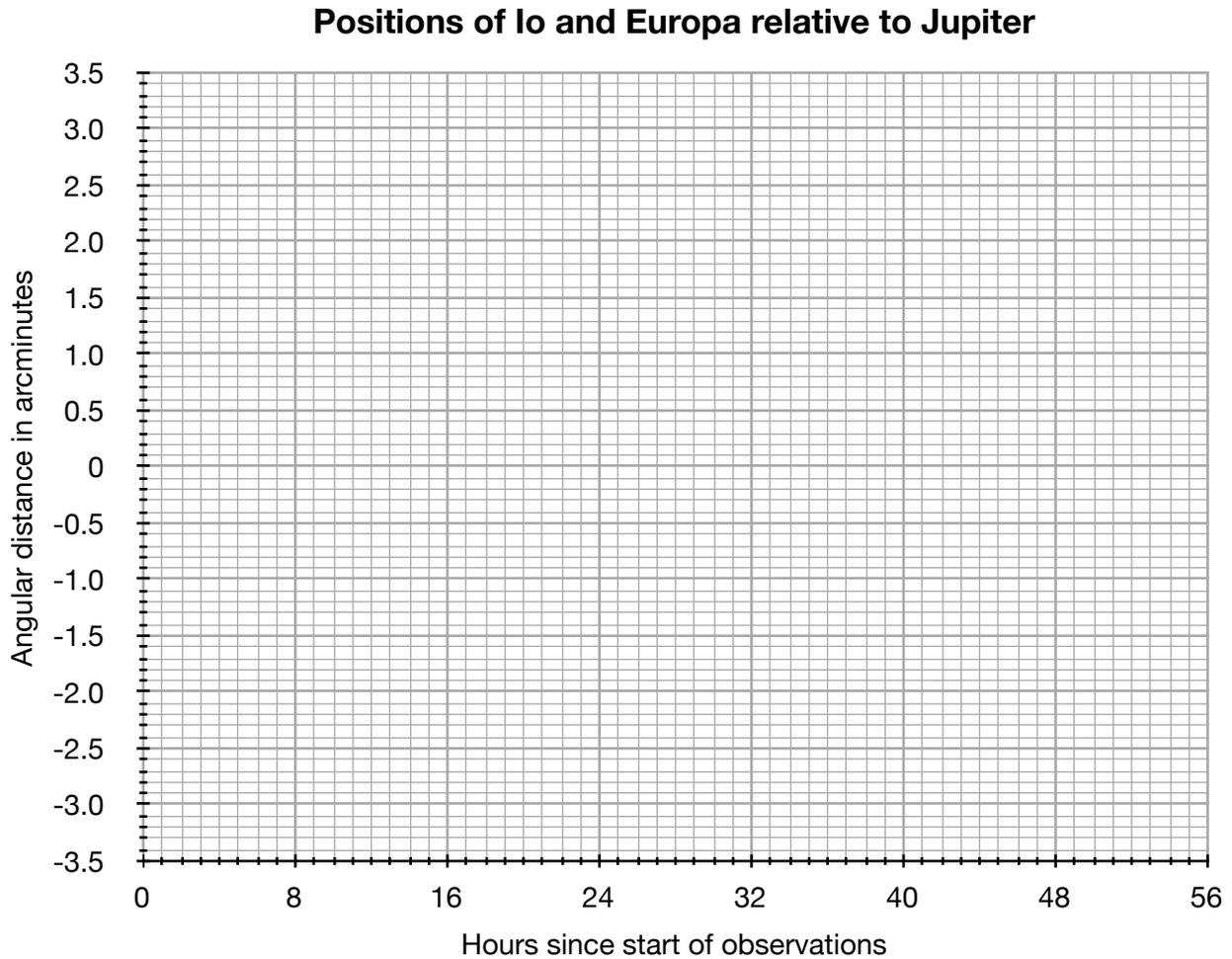
The distance to Jupiter must also be determined to calculate the size of the moon orbits. Historically, this would be determined in A.U. by the Keplerian model of the solar system; for our Stellarium observations, you can read the distance to Jupiter off the top left. Make a note of this distance on 2013-10-25 at 0:00:00; for this lab, we will approximate this as a constant for the measurements considered.

	Hours since first obs.	lo angle (min, sec)	lo E/W	lo angle (decimal min)	Europa angle (min, sec)	Europa E/W	Europa angle (decimal)
10-24 0:00:00	0	1' 52''	E	-1.87'			
10-24 4:00:00							
10-24 8:00:00							
10-24 12:00:00							
10-24 16:00:00							
10-24 20:00:00							
10-25 0:00:00	Distance to Jupiter (A.U.):						
10-25 4:00:00							
10-25 8:00:00							
10-25 12:00:00							
10-25 16:00:00							
10-25 20:00:00							
10-26 0:00:00							
10-26 4:00:00							
10-26 8:00:00							

Analysis of data collected:

In the table on the previous page, convert the measurement to decimal arcminutes using $1 \text{ arcminute} = 60 \text{ arcseconds}$. Label the moon locations to the East of Jupiter as negative and to the West as positive.

Once you have collected the data, mark the positions of the moons on the graph on the following page. Use dots for Io and crosses for Europa to distinguish the two moons, and add a legend explaining which is which.



Question 13: Did Io make a full orbit of Jupiter, returning to its original position, in this time? Did Europa?

The size of the orbits

Question 14: What was the largest angular separation you measured between Jupiter and Io? Of Jupiter and Europa?

Question 15: Could the maximum angular separations be more or less than your largest recorded value? By approximately how much?

To calculate the semi-major axis in A.U., remember the following formula and the recorded distance to Jupiter in A.U.

$$a = d (\theta \text{ in degrees}) \frac{\pi}{180^\circ}$$

To convert to degrees, remember 1 degree = 60 arcmin.

You will later need the semi-major axis in km. Historically, the size of 1 A.U. was determined by timing the transit of Venus from different locations on Earth. The modern value is 1 A.U. = 149,597,870.7 km.

Question 16: What is your best estimate of the semi-major axis of Io in A.U.? What is this in km? Show your work.

Question 17: What is your best estimate of the semi-major axis of Io in A.U.? What is this in km? Show your work.

The period of the orbits

It is easier to determine the period from the graph using the zero-crossing times, instead of trying to figure out exactly when the maximum angular separation was. Smoothly joining the data points before and after the zero-crossing times to estimate the time when the moon crossed the center of Jupiter. **Note that the time between two zero crossings is half of a full period.**

Question 18: What are the zero-crossing times for Io? What is your best estimate of the period of Io in hours? in seconds?

Question 19: What are the zero-crossing times for Europa? What is your best estimate of the period of Io in hours? in seconds?

The mass of Jupiter

Recall the formula for the mass of Jupiter that you found in Question 5.

Question 20: What is your estimate of Jupiter's mass using the orbit of Europa?

Question 21: What is your estimate of Jupiter's mass using the orbit of Europa?

Question 22: Do your answers for questions 17 and 18 agree? Suggest possible reasons for any discrepancies.